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On the galaxy distribution depending on redshift

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Abstract

We compare the observed galaxy redshift distribution with the theoretical predictions from the Standard Cosmological Model (SCM) and the Spherical Expanding Universe Theory (SEUT). We show that the assumption relative to the evolution of galaxy number versus density plays a more important role in the accuracy of these predictions that does the distinct geometrical features of each model.

1. Introduction

In the paper **[Barger et al., 2008]** the observed galaxy redshift distribution N(z) for several thousands of galaxies is shown (Fig. 1).

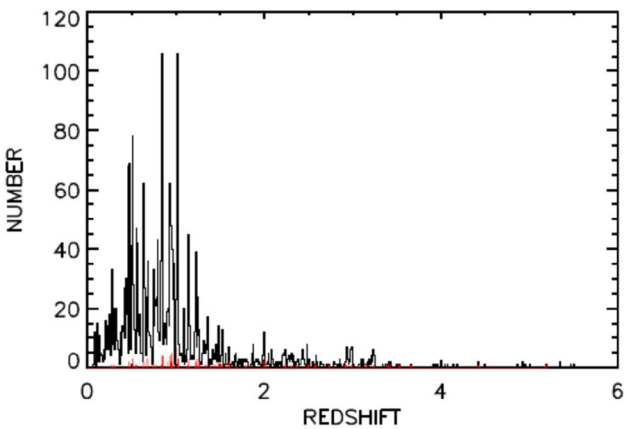

Figure 1. The observed galaxy amount dependence on redshift (**[Barger et al., 2008]**).

The comparison of this empirical data with theoretical predictions from various cosmological models is interesting, because the different models produce different predictions of the N(z) distribution, which allows us to distinguish between them. Indeed, there are many models to choose from, but in this paper, we consider only the expanding flat SCM model, and the expanding spherical SEUT model. Static models are not considered here, since they generally do not imply a redshift[1].

In any *expanding* model, each galaxy count N(z) at a given redshift z corresponds to a set of galaxies that remain equi-distant from an observer even as they all move away from the observer. In other words, all the galaxies are located on the same expanding spherical surface or "shell" (not necessarily a Euclidean one) centered at the observer. (Here we neglect any individual galactic normal motion, so all galaxies on a given observational shell remain on that shell as the Universe expands.) It is clear that the above galaxy count is the product of a *surface numeric density* n(z) and spherical surface area S(z):

$$N(z) = n(z)\cdot S(z)$$

[1] Sometimes a formal correspondence between a distance in the static Universe and redshift is declared as an heuristic principle (see **[Lopez-Corredoira, 2010]**).

When calculating the spherical area S(z), one must take into account two points: the Universal scale factor evolution and the type of its geometry. We will use the dimensionless measure θ of distance between the present-day observer and a galaxy:

$$\theta(t) = \int_{t}^{t_0} d(c\tau)/a(\tau)$$

where t_0 is the present-day Universe age, t is its age at some redshift z, and a(t) is a scale factor that depends on the Universe age, such that $a(t_0) = 1$. One can see that the expression under the integral (in the closed model) is equal to the central angle tangent of the expanding sphere, i.e., in the limit, to the angle itself at a small radius increment. Because of that, one can consider the integral as a total measure of a corresponding angle that is expressed in radians.

Actually, we are interested in the parameter θ(z), not in θ(t). The former depends on the redshift *z evolution as a function of time*. If we know this function, then we can express the surface area S(z) as is shown in Table 1.

Table 1
Bounded surface areas in the different Universe geometry models

Spherical (closed) model	**Flat model**	**Hyperbolic (open) model**
$S \sim [a(z)\cdot\sin\theta(z)]^2$	$S \sim [a(z)\cdot\theta(z)]^2$	$S \sim [a(z)\cdot\mathrm{sh}\,\theta(z)]^2$

In addition, we consider only two hypotheses relative to galaxy number count versus spatial density.

The *first* hypothesis assumes that the *total* galaxy count in the (observable) Universe remains constant during its evolution, i.e., it does not depend on the redshift value. In this case the galaxies are scattering while the scale factor a(z) is increasing, so the volumetric galaxy density is inversely proportional to a^3, and the surface density is inversely proportional to a^2. New galaxies do not appear, or their creation is compensated by the death of others. In this case the galaxy surface numeric density is

$$n(z) = n_0/[a(z)]^2,$$

where n_0 is the present-day galaxy surface numeric density, and a(z) is the scale factor.

The *second* hypothesis assumes that the galaxy *volumetric density*[2] and the surface density are constant), i.e., n(z) = const. This means that while the existing galaxies are scattering (as per Hubble's law), new galaxies are born and fill the empty regions of the Universe.

(Most likely, both of these hypothesis are inexact. We wait for the ultimate answer from future observations and analysis.)

2. The N(z) galaxy redshift distribution in the SCM model

We consider first the standard cosmological model (SCM), in which the Universe is *infinite* and has a *flat* Euclidean geometry **[Wikipedia: Physical Cosmology]**. In such a model, the galaxy redshift distribution function N(z) is proportional to the square of

[2] Note, the assumption that the volumetric galaxy density is constant was applied, for example, in the book **[Klapdor-Kleingrothaus and Zuber, 1997].**

Θ(z), which specifies the distance between the observer and the galaxy. For its calculation, existing computer programs (like http://www.astro.ucla.edu/~wright/CosmoCalc.html) can be used.

On the other hand, the distribution N(z) also depends on the hypothesis relative to the evolution of the galaxy count versus density. If we assume the first hypothesis (the total Universe galaxy amount is constant), then the scale factor does not affect the distribution, because the factor $[a(z)]^2$ is in the denominator of n(z) as well as in the numerator of S(z), i.e:

$$N_1(z) \sim [\theta(z)]^2$$

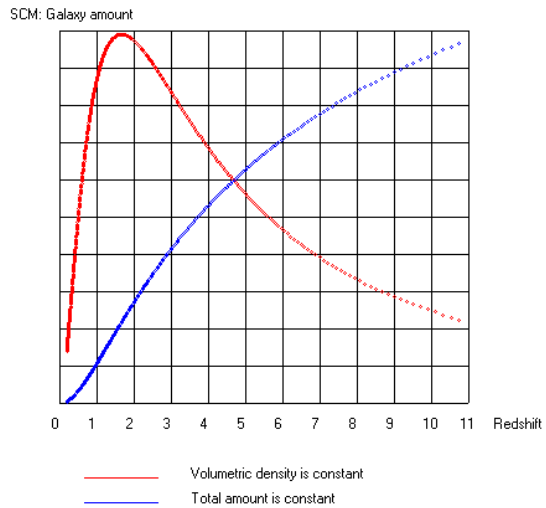

Figure 2.
Theoretical distribution of galaxy counts versus redshift N(z) in the SCM:
blue curve at a constant total Universe galaxy amount,
red curve at a constant volumetric density
(The curves are presented with different vertical axis scales)

But, if we assume the second hypothesis (i.e. the galaxy density is constant), we have to take into account the scale factor a(z), as follows:

$$N_2(z) \sim [a(z)]^2 [\Delta t(z)]^2$$

The results of the calculations are presented in Fig. 2. The first hypothesis corresponds to the *blue* curve while second one corresponds to the *red* curve (the curves are presented with a different vertical axe scale).

As shown in the figure, the SCM N(z) (blue) curve under the constant total galaxy count hypothesis increases monotonically with increasing redshift z (i.e., with the increase in distance between the galaxies and the current-day observer). This increase corresponds naturally to the increasing concentric spherical surface areas in the flat geometry model. Note this predicted curve does not at all correspond to the observed data (Fig.1). In contrast, the SCM N(z) (red) curve under the constant galaxy density hypothesis peaks at z=1.5. This is in qualitative agreement with the observed data (Fig. 1).

3. The N(z) galaxy redshift distribution in the SEUT model

Since 1993 one of us has developed an alternative cosmological model that is particularly described in the paper **[Shulman, 2007a].** This model is called the Spherical Expanding Universe Theory (SEUT). In SEUT, the Universe's evolution is represented by a 4D-cone having a linear generatrix (i.e., linear Universe radius dependent on its age). Any spatial cross-section of the cone at each time moment represents a closed 3D non-Euclidean hyper-surface of a 4D hyper-sphere (Fig. 3). The closure of the cone spatial cross-section geometry is due to the fact that in the SEUT the average matter density is higher than the critical density. Some effective predictions of SEUT, as compared with those of the SCM, are given in the works **[Shulman, 2007]**, **[Shulman and Raffel, 2008]**, **[Shulman, 2010]**.

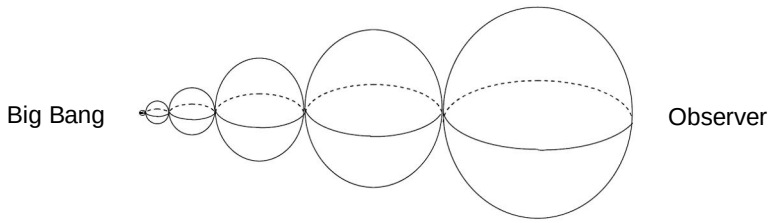

Figure 3.
The expanding close Universe model in the SEUT

Because of the spherical geometry of SEUT, no galaxy redshift distribution N(z) in the model can ever increase monotonically to infinity, since the finite spherical surfaces will impose a maximum peak at some redshift value .

In order to see this we start from the first hypothesis above that the total galaxy amount is constant. Then the Universe size evolution does not affect the redshift distribution N(z). To see this, consider first the similar (but simpler) 2D-problem: to determine the *perimeter length* bounding a part of a spherical surface when the sphere has a constant radius R (Fig. 4).

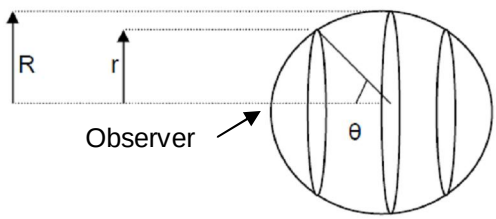

Figure 4.
Perimeter length evolution while one moves from left to right

As one moves away from the observer, the perimeter length $2\pi r$ of the "small" circumference increases first from zero up to maximal value $2\pi R$, and then decreases again to zero. (In contrast, in the SCM Euclidean plane the circumference length increases *monotonically,* and is *unlimited.*) The "small" circumference radius r value depends on the linear central angle Θ (see Fig. 4), where Θ varies from 0 up to π.

In this 2D-analogy the observer is located at the left pole of the sphere while the observational galaxy is located at the *small circumference* specified by $r = R \sin \Theta$. In the SEUT 3D-situation we need to consider the point on a *small sphere* instead of a *small circumference*. If the small circumference *length* in the 2D-case is equal to the $2\pi R \sin \Theta$, then in the 3D-case the small non-Euclidean sphere *area* will be equal to

$4\pi R^2 \sin^2\theta$. At $\theta = \pi/2$ this area is maximal and is equal to the usual 3D-sphere area. In this case, the area values alone determine the galaxy counts that are localized on the sphere corresponding to any given redshift z:

$$N_3(z) \sim \sin^2\theta$$

As was shown in our work **[Shulman and Raffel, 2008]** the angle θ(z) in radians is:

$$\theta(z) = \ln [a(0)/a(z)],$$

where a(z) is the Universe scale factor[3] at a current redshift value z. So, one can build the *red* curve in the Fig. 5 step by step by incrementing z and calculating a(z), θ(z) and area $S \sim [a \sin\theta]^2$ of the small non-Euclidean sphere.

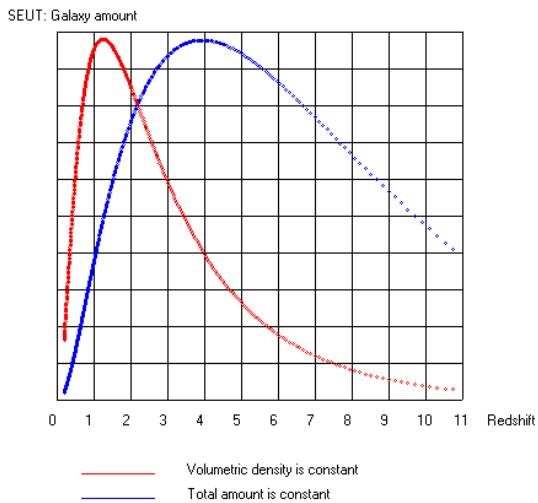

Figure 5.
Theoretical distribution of galaxy count versus redshift N(z) in the SEUT:
Blue curve at a constant total Universe galaxy amount,
Red curve at a constant volumetric density
(The curves are presented with different vertical axis scales.)

But if we assume the second hypothesis in which the galaxy spatial density is constant, then we also have to take into account the Universe size at each given z. This requires that the surface area factor above be multiplied by the square of the current Universe scale factor:

$$N_4(z) \sim a^2 \sin^2\theta$$

The distributions corresponding to both hypothesis in SEUT are presented in Fig. 5. As would be expected, the maximum of the *red* curve (which takes into account the scale factor variation) is smaller (z=1.2) than the maximum of the *blue* curve (z=4). In this case, the red curve maximum is closer to the actual data (Fig. 1) than the blue curve, indicating that the constant galaxy density assumption is preferred within the SEUT.

[3] In the SEUT the scale factor a(z) is proportional to the current Universe age.

4. Conclusion

Our results show that the characteristics of the Universe galaxy evolution (i.e. constant galaxy amount versus constant galaxy density) plays a more important role in the N(z) prediction than the model geometrical features (i.e. flat Euclidean SCM metrics or spherical SEUT metrics).

In this context, the hypothesis that *the total Universe galaxy count* remains constant during its evolution does not produce qualitative agreement between the actual observed N(z) distribution and the predictions from either model. (Note these predictions also differ between themselves).

At the same time, the alternative hypothesis that the galaxy spatial density is constant during the Universe's evolution produces N(z) predictions in both models that are close to the actual observed N(z) distribution.

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