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Entropy of a gravitational force source

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Abstract

I analyze the connection between a gravitational force source and its entropy. The famous **"**Bekenstein bound" or universal entropy bound is generalized.

1. Introduction

In the past century J. Bekenstein established that black hole (BH) should have great entropy proportional to its event horizon. He wrote in [**Bekenstein, 2003**]:

> … a solar mass black hole has an entropy larger [20 orders] than that of a solar mass star which might have been its predecessor. But why should the holes's entropy be the larger by many orders of magnitude? Boltzmann's principle that a system's entropy is the logarithm of the number of microscopic configurations compatible with that system's macroscopic properties, together with the "no hair" principle, suggests that black hole entropy is large because a black hole's aspect cannot tell us precisely which type of system gave rise to it. This extra lack of "composition information" over and above that about specific microscopic configurations may be what makes black hole entropy large. A black hole stands for a large amount of missing information.

The author of the recent famous paper **[Verlinde, 2010]** has shown that the connection between gravitation and entropy can exist not only for BHs, but for "usual" bodies having a state far from gravitational collapse. He introduced an imaginary spherical screen bounded such a body and supporting some holographic information about body mass distribution. He supposed also that gravitation is not fundamental force, so it can be obtained from entopic gradients while the screen radius is changing. However, the assumption on such a "holographic" screen existence supporting a specific information does not seem to be enough well-grounded[1]. On the other hand, the evident connection between gravitation and entropy does not obligatory lead to the entropy's priority. For example, the authors of **[Porcelli and Scibona, 2010]** show that, by using the gravitational dynamics to reproduce the thermodynamic force equation, there is no way to establish the entropic origin of gravity, because the results can be seen the other way around.

In the present paper I argue that the gravitational field can provide the entropy gradients not only for BH, but also in the case of a typical massive body.

2. Field of a central-symmetrical force source and entropy

A mass M creates *central-symmetrical* gravitational field having the potential Φ(r)~1/r . As it is known, the field at a distance r from such a source is determined by the part of the mass located *inside* the sphere having such the radius. Using the approach for BH, we can formulate the statement by another way: *the field at the distance r is generated by equivalent surface gravity σ for the sphere of such the radius, while the same value of σ can correspond to a great number of the real mass configurations inside the sphere*.

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[1] See, particularly, the interesting work **[Myung et al., 2010]**.

In other words, an observer connected with a test particle has always a real uncertainty of the mass distribution, because the interaction between the central source and the particle simply is not physically able to provide more information about it. At a given mass value the uncertainty is depending on the distance between the test particle and the source center. As the gravitational field intensity can be expressed through the equal surface gravity, the entropy corresponding to the sphere surface is equal to the (dimensionless) sphere area.

One can formulate this in terms of thermodynamics. As it is known, a small increment of energy/work ((dW) may be written as the product of generalized force and increment of generalized coordinate. For example, it may be the product of a usual force (e.g., gravity) and displacement (dW=F·dx), or the product of a (gas) pressure and a volume increment (dW=p·dV). But it may also be the product of a temperature (the energy per the surface unit) and a surface increment (dW=T·dA), so, the surface area can play role of entropy.

Let us consider (like Bekenstein) a situation when a test particle falls onto a gravitational field source. At a time moment the particle will transverse an imaginary sphere having some radius that surrounds the source (not black hole in our case). For another test particle outside of this sphere the source mass seems to be increasing due to the first test particle mass accounting. So, the amount of the mass distributions inside the sphere increases too. I.e., the first test particle brings its entropy into the sphere like a situation when a black hole absorbs a particle.

3. Unification of description

Let us to unify the formulas for temperature and entropy that describe both the cases – usual massive body having mass *M* and radius and BH having the same mass *M* and radius $r_G=2GM/c^2$, where *G* is the gravitational constant.

For *the temperature* one can use the Unruh formula (see (**[Good, 2006]**)).

$$T_U = (\hbar/2\pi ck)a$$

where *a* is acceleration, *c* is the velocity of light, *k* is Bolzman constant, *ħ* is Plank constant.

This temperature[2] is just analogy of the Hawking temperature at the BH's event horizon

$$T_H = (\hbar/2\pi ck)\sigma$$

where *σ* is the surface gravity (the gravitational acceleration experienced by a body at the BH's event horizon).

Let us now determine the *entropy* dependence. For the Swhwarzschild black hole the entropy is proportional to the event horizon area. The Verlinde's holographic horizon entropy is also proportional to its area, however, this leads to the fundamental problem which was viewed by Verlinde himself: if the proportionality factor was the same, then the BH's entropy had to be *extremely much less* than an usual body's entropy, because its gravitational radius is much less!

To eliminate this problem I propose to multiply this proportionality factor by the additional ratio (ρ/ρ_{cr}), where ρ is the actual body density, ρ_{cr} is the "critical" density of the collapsed body with the same mass. For example, this ratio for Sun is near 10^{-16}, for Earth it is equal to 10^{-26} (see also Table 1). As it is clear such the ratio effectively increases the body entropy while it approaches to the collapse state. In addition, it

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[2] Its numerical value at the surface of any body in our Universe is between 10^{-15} and 10^{-30} K.

naturally takes into account the direct correlation between the entropy and the mass under imaginary sphere area A. Thus, the proposed formula for arbitrary body (including a BH) entropy S is:

$$S = c^3 A\rho / 4G\hbar\rho_{cr}$$

Note that the area A is proportional to the square of the sphere radius, while the density ρ is inversely proportional (at a given mass) to the radius cube. Hence, finally the entropy is inversely proportional to the radius, i. e., *it rises while the radius decreases*. We are coming to the remarkable result: the mutual attraction process of massive bodies *increases their total entropy*, i.e., corresponds to the natural time evolution due to the second law of thermodynamics.

One can come to the same result while considers the "energetic" aspect: a test particle attracts to a gravitational source and so minimizes the gravitational potentials difference between its current location and the source surface. When the test particle rotates with a constant velocity around the source, then it minimizes the algebraic sum of the gravitational energy and the kinematic one, due to that the rotation occurs at a stationary orbit.

4. Conclusion

So, we come to the next important conclusions.

- For any massive particle the natural entropy gradient along the sphere gravitational radius appears. The more, such a phenomenon can be possible for arbitrary source of a central-symmetrical force. In this sense the connection between a "fundamental" force and entropy is not specific for the gravitational interaction. Because of that the Verlinde's conception does not seem to be valid, in my opinion.
- The proposed generalization of the entropy formula allows us to predict entropy as for usual body state as well as for BH having the enormous entropy at the event horizon. This naturally explains the famous "Bekenstein bound" or universal entropy bound origin, and needed amendments are introduced to the entropy value in all the range of astrophysical objects (see Table 1 below).
- The gravitational forces define the natural evolution direction corresponding to the second law of thermodynamics.

Table 1

Ratio (ρ/ρ_{cr}) for different astrophysical objects

Object	Mass M (kg)	Radius R (m)	Gravitational radius R_G (m)	$(\rho/\rho_{cr}) = (R_G/R)^3$
Earth	$6\cdot10^{24}$	$6\cdot10^{6}$	10^{-2}	$\sim 10^{-26}$
Sun	$2\cdot10^{30}$	$7\cdot10^{8}$	$3\cdot10^{3}$	$\sim 10^{-16}$
Milky Way	$3\cdot10^{42}$	$\sim 10^{19}$	$\sim10^{15}$	$\sim 10^{-12}$
Universe	$\sim 10^{53}$	$\sim 10^{26}$	$\sim 10^{26}$	~ 1

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