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Black Holes and universe evolution

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The global evolution of black holes in our Universe is considered. We also consider a possible evolution of black holes in the universe hierarchy of a differ dimensionality.

1 The growth of black holes in our Universe

The black holes (BHs) *irreversibly* evolve in the frame of some kind of metabolic process: they absorb the energy and matter from external world. The question appears – what is the relation between the growth rate of the *internal* BHs and *our* Universe expansion rate? Can an internal BH obtain sometime a commensurable size or even absorb our Universe?

If the BH's environment is always at a given (constant) density ρ and the external matter specific absorption rate per surface unit remained fixed, then the BH mass growth will be accelerated:

$$dM = S\rho v dt = kM^2\rho v dt$$

Here dM is the BH mass increment, $S=kM^2$ is a BH's surface area which is proportional to its mass M square M ($k=16\pi G^2/c^4$; since $G=7\cdot10^{-11}$ m?s$^{-2}\cdot$kg^{-1} and $c=3\cdot10^8$ m·s^{-1}, then $k=3\cdot10^{-53}$ m$^2\cdot$kg^{-2}), v is average matter fall rate into BH, dt is the time increment. So[1],

$$(dM/dt) = k\rho v M^2$$

It represents the simplest case of the Ricatti equation that has the solution

$$M(t) = \frac{1}{(M_0)^{-1} - k\rho v t}$$

where M_0 is the mass at t=0. It is clear that M(t) infinitely increases at $t_\infty=(k\rho v M_0)^{-1}$ as it is shown on Fig. 1.

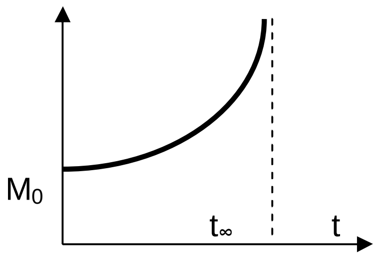

Figure 1. The change of BH mass growth rate

Let us evaluate the evolution time. We set[2] $\rho=10^{-21}$ kg·m^{-3}, $v = 3\cdot10^5$ m·s^{-1} (0.1% of velocity of time), $M_0 = 10^{40}$ kg (a maximal estimation of a supermassive BH mass).

[1] In the work **[Hobbs et al., 2012]** the improved Bondy-Hoyle's formula is recommended that is qualitatively close to our one.

[2] We use the average supermassive BH environment density value pointed in **[DeGraf et al, 2012]**), it is 6 orders more than average matter density in our Universe.

Then we find

$$k\rho v M_0 = (3\cdot10^{-53})\cdot(10^{-21})\cdot(3\cdot10^{5})\cdot(10^{40}) \approx 10\cdot10^{-74+45} = 10^{-28}\ s^{-1}$$

and

$$t_\infty = (1/k\rho v M_0) = 10^{28}\ s \approx 10^{21}\ \text{years}$$

This value is immeasurably more than the Universe actual age (10^{10} years). It is clear that some orders data error cannot substantially change the result, so any catastrophe threat to the mankind is absent.

Since the value (t/t_∞) is small we can approximately write

$$M(t) \approx M_0\cdot(1 + t/t_\infty)$$

Thus, the BH mass increment changes now over time as

$$\Delta M(t) \approx M_0\cdot t\cdot(k\rho v)$$

As we can see, in the first approximation the BH's growth is proportional to the environment density ρ. Namely this reason could explain the fact that the astrophysicists reveal everywhere BHs *in the center of galaxies*, because just there the value of ρ is some orders more than in other regions of the Universe.

Let us note the important fact. The BH irreversible growth occurs just while it is accompanied by its entropy (and its surface area) increase. In the flat Euclidean space this quantity can increase infinitely. However, if our 3D space is closed (i.e. it has the spherical type of metrics), then the BH surface area growth happens simultaneously with its mass increment just before the "equatorial" size of the Universe is obtained. After that a new situation appears: the BH mass growth has to lead to the BH surface area and entropy decreasing (not increasing)! But this contradicts to the Second Law of Thermodynamics, because of that the BH growth has to stop. *Finally, the Universe probably will consist in two great BHs that cannot merge*.

2 Black holes in a universe of a differ dimensionality

In the publications **[Shulman, 2009, 2011a, 2011b, 2011c]**) we developed our Universe representation as the black hole inside of some 4D hyper-universe that *linearly* expands in its own time, so its age remains exactly proportional to its actual size and mass. Our Universe itself has the dimensionality n_u=3 while the hyper-universe has the dimensionality n_h=4, i.e. n_u=n_h-1. In return, as we believe, the BHs inside of our Universe have lower dimensionality, they represent 2D spherical surfaces (this corresponds to the common BH "membrane" approach for an external observer). Thus, a BH hierarchy appears having interesting relations between the levels of the hierarchy.

Let us consider how BH's geometrical radius R and gravitational one R_G depend on its mass M in n-dimensional space (note, such the approach is not applicable for one-dimensional case).

It is clear that the BH *geometrical* radius R in n-dimensional space can be expressed (at a given density) through its mass M as

$$R \sim (M)^{1/n}$$

(e.g., in our 3D space we have $R \sim (M)^{1/3}$).

Further, we accept the hypothesis that in n-dimensional space the gravitational interaction forces decrease with distance (as the Gauss theorem states) as R^{1-n} (particularly, as R^{-2} in our 3D Universe). This hypothesis is motivated for 2D surface by electrostatic cylindrical (not spherical) analogy: in this case the field intensity decreases as R^{-1}, and electrostatic potential φ decreases as $ln\ (R_{max}/R)$, where R_{max} is the radius of an external cylindrical capacitor plate. Analogously, the law $\varphi \sim M\ ln\ (R_{max}/R)$ describes a static gravitational field on 2D surface if R_{max} is large enough.

In a n-dimensional space (where n>2) the potential will change as R^{2-n} (in our 3D Universe the potential will change as $\varphi \sim M/R$).

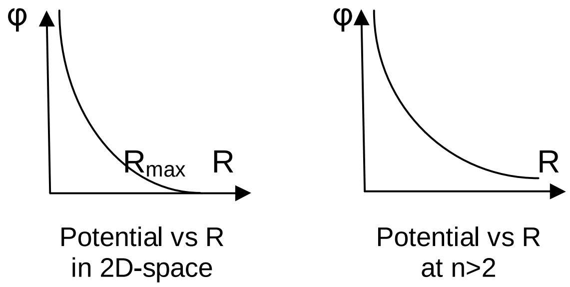

Figure 2. The potential evolution vs distance

The BH gravitational radius R_G in classical physics of 3D space can be found from the parity condition between specific (per mass unit) kinetic energy and the gravitational potential of a source having the mass M: $c^2/2= \varphi(R_G)$, where c is velocity of light. Using the same approach for 2D configuration we have

$$c^2/2 \sim G_2M\cdot \ln(R_{max}/R_G)$$

from where

$$R_G \sim R_{max} \exp(- c^2/2G_2M)$$

Here G_2 is the Gravity constant in 2D space. In general case we have the following table where constants G_2, G_3, G_n differ at least by their dimensionality.

Table 1

Relationships for potential and gravitational radii

	n=2	n=3	n>3
Source equality for potential	$c^2/2= G_2M \ln(R_{max}/R_G)$	$c^2/2= G_3M/R_G$	$c^2/2=G_nM/(R_G)^{n-2}$
Expression for R_G	$R_G= R_{max} \exp(- c^2/2G_2M)$	$R_G=2G_3M/c^2$	$R_G=[2G_nM/c^2]^{1/(n-2)}$

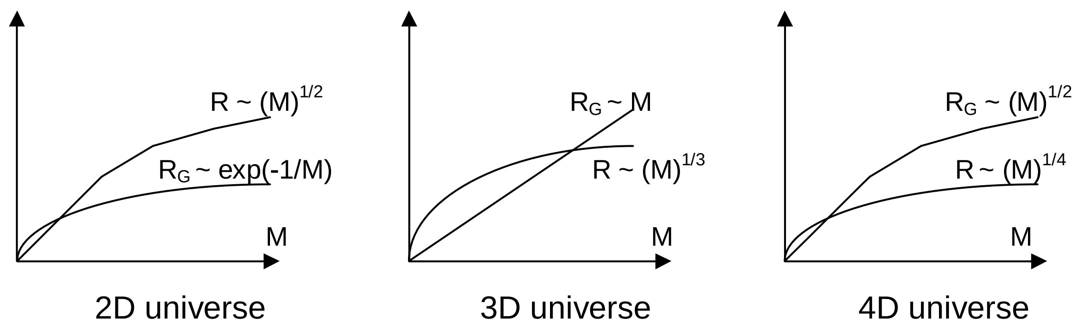

Figure 3. Evolution of BH geometrical R and gravitational R_G radii vs BH mass M

In the Fig. 3 we compare the BH radii R and R_G dependence on BH mass M at a given density. From this Figure it follows that at some BH mass (at a given density) the relation between R and R_G inverses. Indeed, at n>2 we have $R > R_G$ at left from the crosspoint, and $R < R_G$ at right from the same point.

This means that in our Universe (and in the universes of higher dimensionality) the small material bodies (at a given density) are in the normal state. However, they collapse and transform to BH when their mass and size become a critical ones (at a same density). Consequently, their size has to be *finite*.

Contrary, at n=2 the material objects should evolve by inverse way. Perhaps, it follows that such the evolution is impossible in principle.

3 Conclusion

Thus, we come to the following conclusions:

- Our Universe cannot be infinite and is black hole in some external 4D hyper-universe[3]. Analogously, a hierarchy of finite black holes of higher dimensionalities has to exist.
- Since BHs exist only inside of some “maternal ” BH having a closed (spherical) geometry they can grow only up to half of this maternal BH, and because of that never can absorb it. Whatever happened inside of it, any information about its structure will not accessible to an external observer.
- The global evolution of BHs in our Universe finally will lead rather to creation of two giant ones that will occupy practically all the Universe.
- The “daughter” BHs emergence inside of 2D BH (that in return represents an internal BH in our 3D Universe) seems to be impossible or problematical.

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[3] See **[Shulman, 2009, 2011a, 2011b, 2011c].**