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"Extremely Large" Dirac Numbers and Fundamental Constants in Cosmology

The Dirac's approach to interpret several dimensionless physical constant combinations is revisited. Particularly, I argue that ratio of the current Universe mass to its size is constant, and the ratio of this mass to the Planck mass (as well as the ratio of the current Universe size to the Planck length) is typical "extremely large" Dirac number that specifies the dimensionless Universe age. I suppose the new viewpoint on the Planck action constant.

The Dirac's conjecture

The great British physicist Paul Dirac in his lecture presented at the School of Physics, University of New South Wales Kensington, Sydney, Australia in August of 1975 **[Dirac, 1978]** pointed out to the interesting coincidence between the value orders of several dimensionless combinations of physical quantities. For instance, the ratio of the electrostatic force to the gravitational one is

$$\frac{e^2}{Gm_e m_{prot}} \approx 2 \cdot 10^{39}$$

where e is the charge of the electron (and proton), G is the gravitational constant, and m_e , m_{prot} are the mass of the electron and the proton, c is the velocity of light. On the other hand, the ratio of the current Universe age τ_U to the time required for light to traverse a classical electron $\tau_e \approx e^2 / m_e c^3$ is:

$$\frac{\tau_U}{\tau_e} \approx 7 \cdot 10^{39}$$

Dirac supposed that "there must be some fundamental reason in Nature why these two large numbers should be so close together", that both of them should increase with time, as well as the gravitational constant and Universe mass also should evolve with time. This conjecture didn't lead to a successful prediction, it was considered by many physicists as some kind of numerology. The problem itself remains to be interesting for the researchers; however, it was not commonly resolved.

Even the genius failure shouldn't be ignored because the treasure that he didn't find may be concealed very closely! This publication represents an approach to find some rational (and close) explanation of the above facts in the framework of the cosmological model that differs from the Standard Cosmological Model (ΛCDM). I develop that nonstandard model (the Spherical Expanding Universe Theory – SEUT) since 1993 (see, e.g., **[Shulman, 2011]**). The SEUT postulates that our Universe is the *black hole in an external 4D super-Universe*. It is the reason of the Universe irreversible expansion due to the matter and energy absorption. Correspondingly, in our model (contrary to the Standard Model), the Universe size evolution is accompanied by its mass increase, and the conservation law also cannot be fulfilled (however, this effect is *relatively* very small: ~10^{-10} per year in the current epoch) because the Universe is the open system [1].

Planck units

In 1899 Max Planck proposed the system of the "measurement natural units". It bases now particularly (but not exclusively) on the velocity of light c, the gravitational constant G and the Planck constant $\hbar$.

In this system one may determine the Planck units of mass, length, and density using the expressions

$$m_p = \sqrt{\frac{\hbar c}{G}} \approx 2.1 \cdot 10^{-5} g\,, \qquad l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \cdot 10^{-33} sm\,, \qquad \rho_p = \frac{m_p}{l_p^3} \approx 10^{94} g / sm^3$$

Note that the ratio of two first above quantities is equal to

$$\frac{m_p}{l_p} = \sqrt{\frac{(\hbar c / G)}{(\hbar G / c^3)}} = \frac{c^2}{G} \approx 10^{28} g / sm$$

One also may introduce the Planck temperature (k_B is the Boltzmann constant):

$$T_p = \frac{m_p c^2}{k_B} = \sqrt{\frac{\hbar c^5}{G (k_B)^2}} \approx 10^{32} K$$

Physical meaning of the gravitational constant

Any dimensional physical constant is connected with physical meaning of the measured quantities where it is used as well as with an arbitrary choice of the unit system. The gravitational constant G analysis one usually associates with Newton gravity law:

$$F = G M_1 M_2 / r^2$$

However, as I believe, it is better to use another known relationship:

$$R = 2GM / c^2$$

where R is the gravitational radius of a body having mass M, c is the velocity of light. The simple linear connection between distance and corresponding mass follows from this relationship, so we may consider them as the same physical quantity.

Moreover, from the point of view of an external observer the above relationship is "the basic equation of state of the black hole", because its size increases with mass, and their ratio has to be constant. Let us denote the Universe current radius as R_U and

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[1] A.D. Linde in his lecture in Physical Institute Of the Russian Academy of Sciences (2005) talked that the initial mass of the Universe was extremely small – less than one milligram. However, the SEUT and the modern inflationary cosmology not only drastically differ one from another but even compete in order to explain several difficulties of the Standard Model. I thank Yu. A. Lebedev who said me about this fact.

its current mass as M_U. If the main postulate of the SEUT is true and our Universe is really the black hole, then the ratio of its mass to the radius at any epoch (including modern one) is equal to the constant quantity

$$\frac{M_U}{R_U} = \frac{c^2}{2G}$$

As it is well known, $M_U \approx 10^{56} g$, $R_U \approx 10^{28} sm$, so their ratio has the order near 10^{28} *g/sm*.

“Extremely large” Dirac number

Thus, if we ignore the unimportant factor 2 (one may exclude it by the redefining m_p or l_p), then we get

$$\frac{M_U}{R_U} = \frac{m_p}{l_p} = \frac{c^2}{G}$$

This fact confirms the main conjecture of the SEUT that the Universe mass/size ratio is unchanged at any epoch.

Furthermore, we now can easily deduce and check **[Wikipedia]** that

$$\frac{M_U}{m_p} = \frac{R_U}{l_p} \approx 10^{60} \equiv D$$

The introduced number D shows how much the current Universe mass and size are increased during the Universe evolution time and determines the dimensionless Universe age. I propose name it “the Dirac number”.

It is naturally to believe that Planck time is the time point of the Big Bang, i.e. the time point of our Universe ancestor gravitational collapse. Then we come to the following remarkable issues:

- The initial size of our Universe was $l_p \approx 10^{-33} sm$.
- The initial mass of our Universe[2] was $m_p \approx 10^{-5} g$.

One should point out that the characteristic modern and Planck durations can be found using simple division of the corresponding spatial sizes by the velocity of light, so the ratio of these durations is trivially equal to the ratio of these sizes.

Let us note that in the Standard Cosmological Model there is so-called cosmological constant Λ that was introduced into Einstein equations as “free parameter”; although that constant seems to solve some problem, however, instead of that it creates other problems that are more difficult than previous one. In the SEUT one has not to introduce this constant because in it the solution automatically appears where the matter (negative) pressure[3] is inversely proportional to the Universe current size.

[2] One sometimes can see in the literature the incorrect (in my opinion) statement that Planck mass is “the maximal possible mass of an elementary particle”.

[3] One usually supposes (following Einstein) in the Standard Model that the matter pressure in the Universe is zero.

Finally, the *fictive* cosmological constant that physically corresponds to this current density turns out to be equal

$$\Lambda \approx \frac{1}{R_U^2} = \frac{1}{D^2 l_p^2} \approx 10^{-122} \frac{1}{l_p^2}$$

Physical meaning of the "Planck constant"

Thus, the ratio $\frac{M_U}{R_U} = \frac{m_p}{l_p}$ turns out to be not depending on the Planck constant $\hbar$. However, the relation between m_p and M_U, and also between l_p and R_U, depends on it. Probably, the parameter $\hbar$ somehow specifies the current epoch of the Universe expansion and can have another value at other epoch (cf. the above conjecture of Dirac!).

Earlier I showed **[Shulman, 2004]** that the basic novelty of Heisenberg when he created the Quantum Mechanics consisted in the introduction of the *complex* numbers to describe (particularly) the electron spatial orbits in atoms. The *new commutation rules* for such observables as position and momentum (or time and energy) was the straight consequence of this introduction; in the classic physics these rules were the trivial ones for usual c-numbers. In particular, when one describes classical oscillators using the *complex* numbers (which do not equal to their complex conjugates) the analogs of the quantum-mechanical commutators appear. Of course, the quantity $\hbar$ is not used there, but the expressions[4] like Ψ, Ψ^*, $|\Psi| = \Psi^*\Psi$, etc. do appear. Each such the oscillator without loss (phase shift between position q and momentum p is $\pi/2$) can be described using a commutator in a form

$$[q, p] = q^* p - qp^* = iq_{\max} p_{\max} \quad ,$$

where the maximal (real) values of q and p are on the right hand.

The genial achievement of Dirac that led to the transition to the matematics of Quantum Mechanics finally consisted in the replacement of the individual products $q_{\max} p_{\max}$ for each concrete classical oscillator by the *universal* dimensional constant of action[5].

Note that when we talk on an oscillator we in fact take into account the wave origin of matter as de Broglie declared. Also we have to account that the wave which corresponds to quantum particles ("de Broglie - Bohm Pilot Wave") is *non-local* and *propagates* in space with infinite velocity. These waves cannot transfer physical information, but they can lead to a non-local correlation between spatial points that are separated by a timelike distance (like EPR experiments).

As it is well known, the parameter $\hbar$ appears in the quantum relations where one operates by *energy* and *momentum*. In the framework of the physics geometrization I would like to precise what physical meaning these quantities have. The SEUT proposes the simple and clear statement that completely corresponds to the Quantum Theory:

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[4] The simplest well known example of such the conception use is the power calculation in the alternative current circuits.

[5] Dirac he-self originated from the Poisson bracket that is very closed to such the complex commutator for a classical oscillator.

Energy and momentum are the quantum numbers that express the ratio of the Universe age (radius) to de Broglie wave period (wave length) of the concrete particle.

So, the wave nature of every particle makes it a specific “clock” and “ruler” that measure the Universe evolution. Let us remember that in the SEUT (contrary to the Standard Model) de Broglie period and length are considered as unchanged over the Universe expansion.

As it is shown, the Schrödinger equation historically was deduced by its author via the generalization of the expression for the de Broglie wave length. Let us make an inverse way - from the particle potential well to the free one (according with the Seut we consider the Universe size as *finite*).

Let us now mentally increase the well width (in which some quantum particle is disposed) until one border of the well will coincide to the other one (remember, the Universe is considered as finite and closed). Now we don’t need in the infinitely high walls of the well as the condition to quantize, and the length’s number has to be integer again. Meanwhile, the wave length expresses now via the Universe perimeter, not via an arbitrary well size.

As we didn’t change something in the formal problem definition, the product wave length and momentum is stll equal to the Planck constant. It is very important and nontrivial (but predictable) that appearing discretness occurs to be the simple consequence of the Bohr quantization rule. So, the fundamental link between momentum and wave length turns out to be the same for photons and other particles having a non-zero mass:

$$p = 2\pi\hbar/\lambda \ , \quad E = 2\pi\hbar/T$$

Here p is a particle momentum, E is a particle energy, λ and T are particle de Broglie wave length and wave period. Accorging with the SEUT (and contrary to the Standard Cosmological Model) λ and T remain unchanged, and momentum p and energy E increase linearly with time. Thus, such the linear rise is due to the quantity $\hbar$ that has now to be proportional to the size (age) of the Universe and should not be called the “Planck constant”:

$$\hbar(t) = \Pi \cdot R$$

(To retrieve this we will denote the new constant by the Greek letter “Π” in honor of Planck). So, the new constant may be found from the relationship

$$\Pi = \frac{\hbar_U}{R_U} \approx \frac{10^{-27} erg \cdot s}{10^{28} sm} = 10^{-55} g/s$$

All the quantum processes should be considered in the framework of the SEUT as *non-local* oscillations of whole Universe (like shell oscillations of the hypersphere).

When the Universe size will, e.g., twice, then Planck value will multiply by 2 too. On the other hand, when the Universe size was equal to $l_p \approx 1.6 \cdot 10^{-33} sm$, the Planck parameter was $\hbar_p = \Pi \cdot l_p = 10^{-55} \cdot 1.6 \cdot 10^{-33} \approx 10^{-88} g/s$, so $\hbar_U / \hbar_p \approx 10^{60} = D$. I don’t see any decisive objections against these drastical conclusions.

The proposed viewpoint allows us to estimate the particle minimal mass value at the current epoch. According to de Broglie rule it has to correspond to the maximal size (the Universe size) R_U:

$$m_{\min} c \approx \frac{2\pi\hbar}{R_U}$$

The found minimal mass value is very small, ~ 10^{-66} *g*, while the electron mass is near 10^{-27} *g*. Note, the close value $m_{\min}$ was found in **[Wesson, 2003]** using a dimensionality consideration.

Electrical charges

It is interesting to compare the Planck mass with the Planck charge:

$$m_p = \sqrt{\hbar c \cdot \frac{1}{G}} \quad \text{и} \quad q_p = \sqrt{\hbar c \cdot 2\varepsilon_0}\ .$$

Here ε_0 is the permittivity of free space. Such the comparison leads to the conclusion that the ratio $q_p / m_p = \sqrt{2\varepsilon_0 G}$ doesn't depend on time. Because of that it is possible that electric charges (e.g., electron charge) increase proportionally to the Universe age, not only masses. If so, then the fine structure constant $\alpha = (e / q_p)^2$ increases proportionally to the Universe age too.

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