Usual collapse and unusual one

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Abstract

Some phenomena emerging while one approaches immediately to the collapse of a sphere that has a finite size and consists in perfect fluid are considered.

1. Introduction

An analyzis of the gravitational collapse is often based on the assumption that a collapsing object has a real size (much) less than its gravitational radius (i.e. is considered as a material point). This can lead to the loss of several important and very interesting details and may become inapplicable.

We use the Schwarzschild's solution for the material uniform sphere problem **[Tolman, 1934]**. Such a sphere has a radius $r_1$and a density ρ, and consists in perfect fluid. The analytical description is represented in the Appendix while the main text uses the graphical dependencies on a current radius r. The dependencies are specified by a curvature radius R (or by a corresponding density value) as parameter.

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2. Evolution of the metric tensor component G_{00}

In the General Relativity (GR) the metric tensor component G_{00} is like to the gravitation potential in the Newton's gravitation theory. Let us consider this dimensionless quantity as it approaches to the collapse. Graphically it is presented in the range $0 \le r/r_1 \le 1.1$.

If the situation is far from the collapse (the measure is specified by a ratio R/r_1 or ρ/ρ_0), the curve increases monotonically (see Fig. 1).

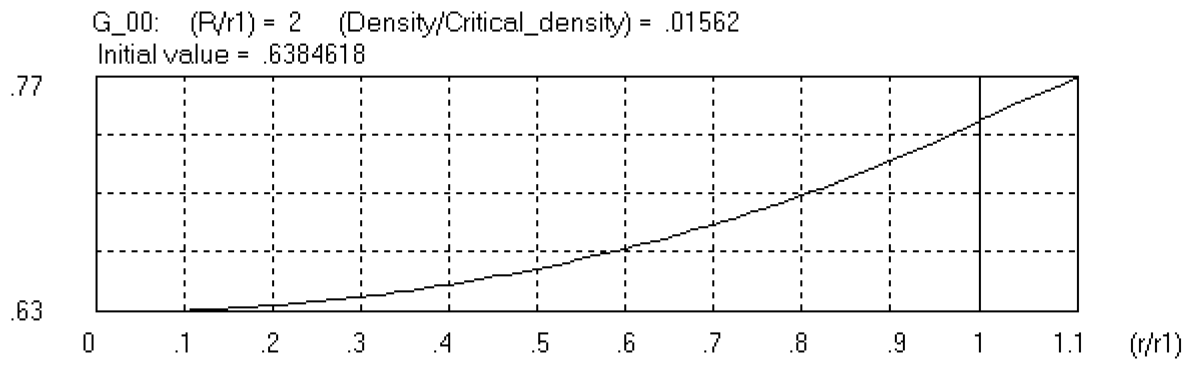


Figure 1. The distribution of G_{00} far from the collapse

However, as one approaches to the collapse (at $1 \le R/r_1 \le 3/2\sqrt{2}$) the monotonic behavior of the dependence is changing: the additional extremum appears (Fig. 2). The initial part of the curve "bulges" up having a maximum at the center of the sphere, while a new minimum appears inside of the sphere. When one approaches to the collapse, the minimum drifts to the sphere bound.

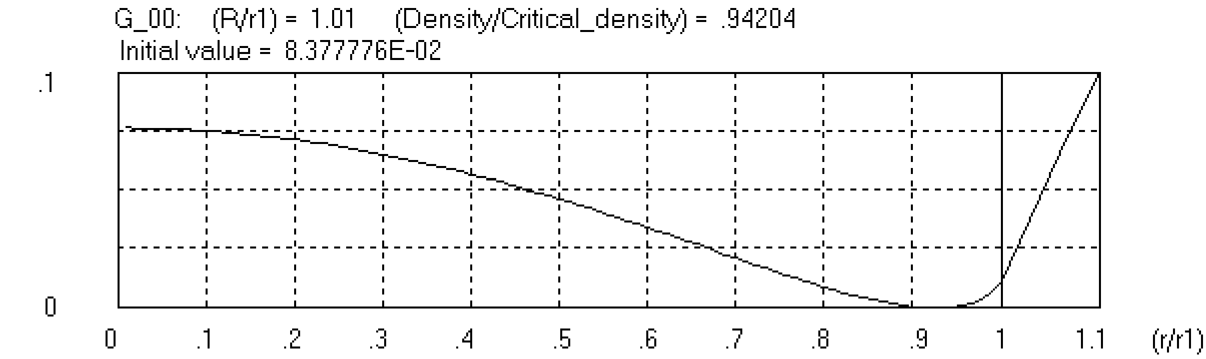


Figure 2. The distribution of G_{00} when the additional extremum appears

Finally, at the collapse state this minimum transforms to the jog localized just on the sphere bound (Fig. 3).

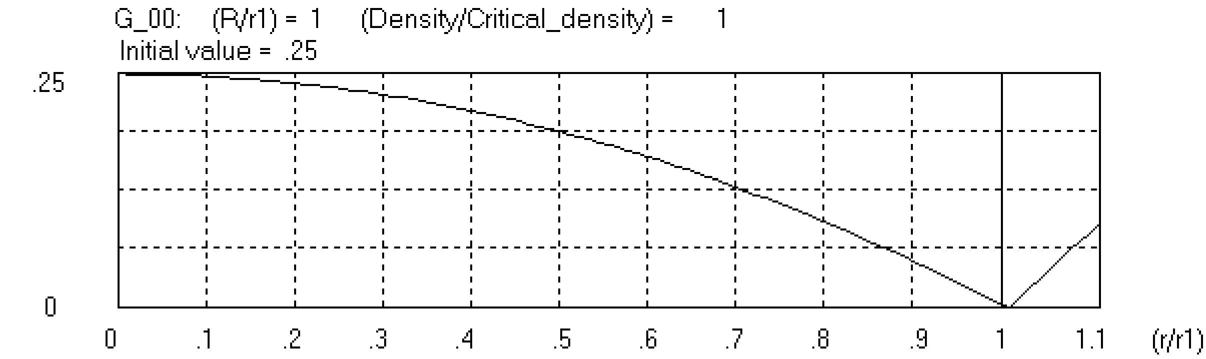


Figure 3. The distribution of G_{00} when the collapse is realized

3. Evolution of the pressure P inside the sphere

Let us consider now the pressure's behavior. It strictly positive far from the collapse and decreases down to zero monotonically from the center of the sphere to the bound (Fig. 4).

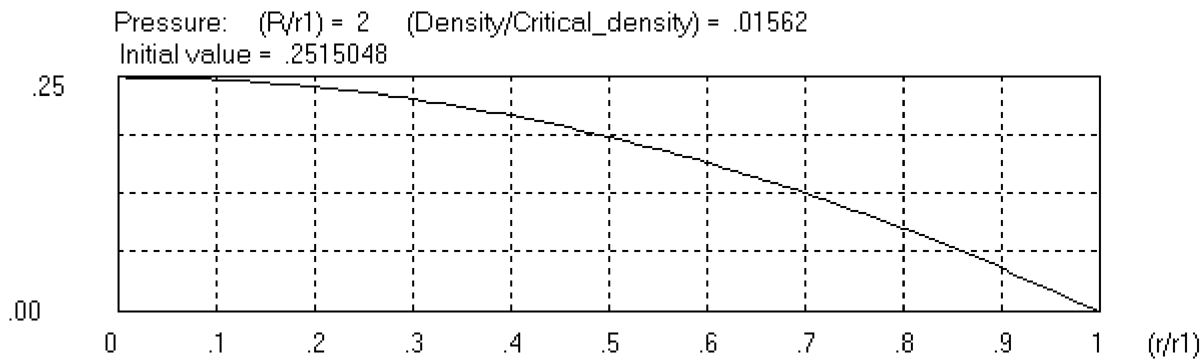
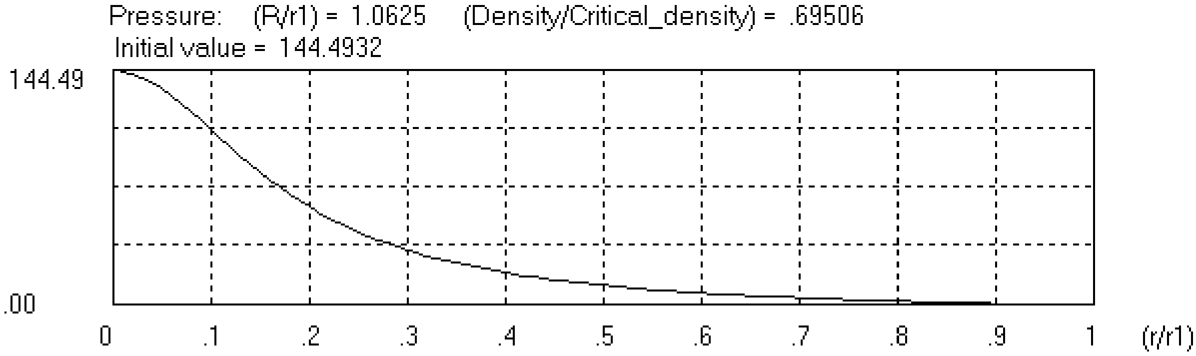
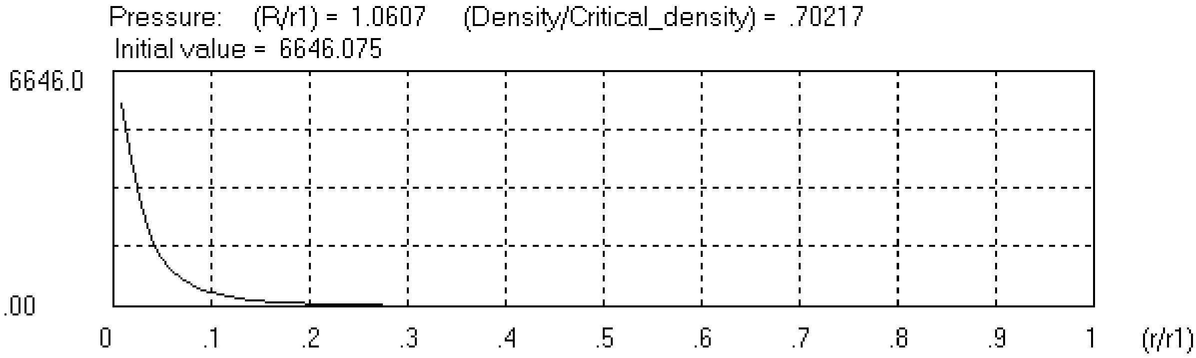


Figure 4. The distribution of the pressure far from the collapse

As well as one approaches to the domain of the additional extremum component G_{00} existing the pressure distribution irregularity increases as it was "flattened" at the central region of the sphere (Figs. 5a and 5b).



a)



b)

Figure 5. The distribution of the pressure near collapse
when one extremum (minimum) of the G_{00} exists only

The very different situation is observed more close to the collapse after appearing the second G_{00}'s extremum, i.e., at $3/2\sqrt{2}>(R/r_1)>1$. The "unidirectional pulse" of the pressure having a finite value transforms to the bidirectional break of the function, its left side (before the break) becomes to be negative, while the right side (after the break) becomes to be positive (fig. 6a). At the break point the function value is not defined (the value is $\pm\infty$).

So, yet before collapse, but near its boundary $R/r_1 = 1$, we observe a new phenomenon, which consists in negative pressure. This one does not represent something unknown for physics. A pressure is positive at usual conditions, i.e. it is directed as a body was "compressed" and hence is going to enlarge. However, it is not obligatory, a body may also be in a state with a negative pressure. In such a state a body seems to be "extended" and going to compress. For example, a superheated liquid can be at a negative pressure; such a liquid operates to its boundary surface with a force directed inside of its volume. In our case the negative pressure may be due to a volume "enlarging", because the metrics changes.

In this range of conditions, while one approaches to the collapse mode the break point position is clearly shifting to the bound of the sphere from its center. The initial pressure is negative and approaches to the value –3, and the bound pressure is always zero. At the collapse mode ($R/r_1 = 1$) the pressure at the sphere bound becomes to be unidentified (0/0).

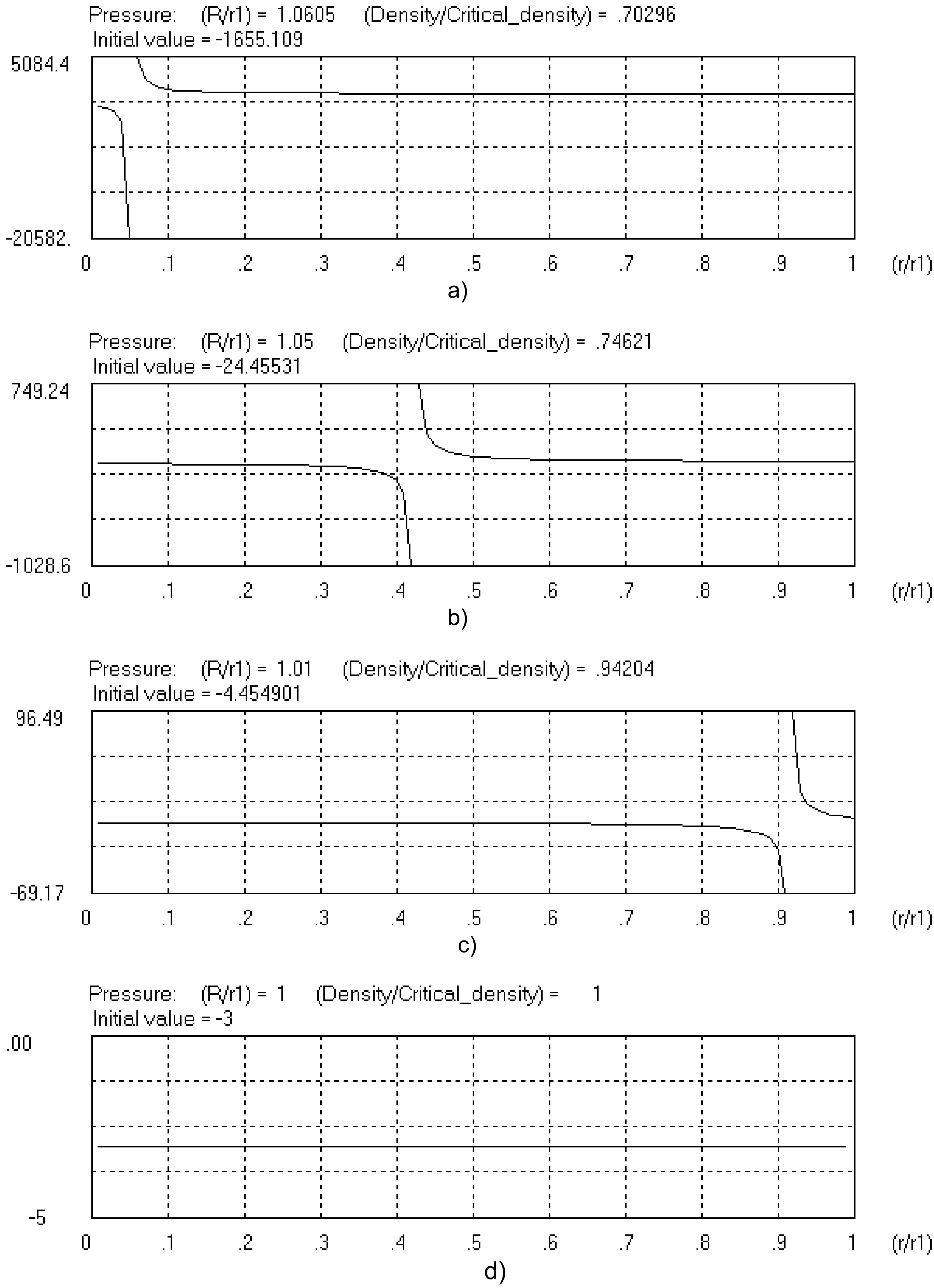


Figure 6. The transition from a finite unidirectional pressure pulse to the noncontinuous behavior after the second extremum (minimum) of the component G_{00} appears. The shifting of the pressure break point to the sphere bound while one approaches to the collapse mode.

4. Conclusion

In the present work we have seen that the collapsing object real size account allows us to reveal some new and important details of this physical phenomenon. Particularly, when a real body size is a little more than its gravitational radius, the metric tensor component G_{00} plot gets the additional extremum that transforms to the to the jog localized just on the sphere bound when the both sizes become equal. The pressure remainds to be zero outside from the sphere and close to the collapse obtains an infite bipolar rupture. While one approaches to the collapse condition, this rupture is displacing beyond the sphere bounds.

It should be noted that the matter pressure plays a very important role in this model while in the model of a point mass collapse one does not account this role at all. This difference is important when one analyzes the cosmological problem (see **[Shulman, 2007]** and **[Shulman, 2006]**).

Appendix

1. Basic Relationships

The book **[Tolman, 1934]** describes the Einstein's equation solution that was found by Schwarzschild in the metrics

$$ds^2 = G_{00}\, dt^2 - r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2) - dr^2 / (1 - r^2/R^2).$$

One means that a sphere consisting in the perfect fluid having a density ρ has a radius r_1 and is surrounded by an "empty" space.

The Schwarzschild's solution represents two functions: metric tensor component $G_{00}(r)$ (that corresponds to the Newton's gravitational potential) and pressure P(r). In this solution one uses the sphere radius r_1, a current radius r and the curvature radius R:

$$R^2 = 3c^2/ (8\pi G\rho)$$

Since the gravitational radius R_G of such a sphere is

$$R_G = 2GM/c^2 = 2\, (4\pi r_1^3 G\rho)/(3c^2) = r_1^3 / R^2$$

then R is univocallly connected with the gravitational radius R_G and a the sphere radius r_1 by the ralation

$$R_G/r_1 = (r_1/R)^2$$

Also, since at the collapse we have $R_G = R_G = r_1$, then

$$1 \geq (\rho/\rho_0) = R_G^2/R^2 = (r_1^6/R^6)$$

where ρ_0 is the density corresponding to the collapse (i.e. to the condition $R_G = r_1$).

The The Schwarzschild's solution is given by the dimensionless relationships

$$G_{00} = \left(\frac{3}{2}\sqrt{1 - r_1^2/R^2} - \frac{1}{2}\sqrt{1 - r^2/R^2}\right)^2$$

and

$$(P/P_0) = \Phi(r, r_1, R),$$

where $P_0 = \rho_0 c^2/3$, and the function $\Phi(r, r_1, R)$ is

$$\Phi(r, r_1, R) = \frac{3\sqrt{1-(r/R)^2} - 3\sqrt{1-(r_1/R)^2}}{3\sqrt{1-(r_1/R)^2} - \sqrt{1-(r/R)^2}}$$

2. Specification of the evolution of $G_{00}(r, r_1, R)$

It is easy to see that G_{00} is always non-negative and its initial is determined by

$$G_{00}(r_1 = 0) = 9(1 - r_1^2/R^2) / 4$$

Furthermore, its derivative is equal to:

$$(dG_{00}/dr) = \left(\frac{3}{2}\sqrt{1-r_1^2/R^2} - \frac{1}{2}\sqrt{1-r^2/R^2}\right) r / \left(2R^2\sqrt{1-r^2/R^2}\right)$$

At $r = 0$ we have $(dG_{00}/dr) = 0$, and at $r = r_1$ (more precisely, at $r = r_1 - 0$) we find

$$(dG_{00}/dr) = r_1/R^2$$

It is well known that there exists usually one extremum (minimum at $r = 0$) of the function $G_{00}(R/r_1)$. However, it turns out that *before* collapse the function behavior changes, and the additional extremum appears. In fact, the equation $(dG_{00}/dr) = 0$ is equivalent to the condition

$$9(1 - r_1^2/R^2) = 1 - r^2/R^2$$

or

$$r/R = \sqrt{9r_1^2/R^2 - 8}\,.$$

Hence, the additional real extremum appears in the condition range

$$3/2\sqrt{2} > (R/r_1) > 1$$

In this case the second internal extremum becomes to be mimimum, and the first minimum at $r = 0$ is now the local minimum. This internal minimum[1] transforms at the collapse to the jog of G_{00} localized just on the sphere bound, when the derivative changes its sign, i.e., $(dG_{00}/dr) = - r_1/R^2$. Really, in the external region the expressions under all radicals will become negative ones, so finally the derivative will be multiplied by the imaginary unit square (–1).

The initial value (at $r = 0$) of the component G_{00} is always

$$G_{00\,\text{init}} = \left(\frac{3}{2}\sqrt{1-r_1^2/R^2} - \frac{1}{2}\right)^2$$

and a boundary one (at $r = r_1$) is

$$G_{00\,\text{bound}} = 1 - r_1/R$$

[1] It is interesting to compare this fact with the statement from **[Novikov and Frolov, 1989]**: "The condition $r = 3R_G$ corresponds to the critical circular orbit that separates a stable motion from an unstable one. … At this the system energy is $E = \sqrt{8/9}$ …"

3. Specification of the evolution of $P(r, r_1, R)$

In order to analyze the function Φ(r) behavior it is suitable to represent it in a more compact form

$$\Phi = 3(x - z)/(3z - x) = - 3(x - z)/(x - 3z),$$

where x is first radical of the numerator depending on the variable (r/R), and z is second radical depending on the parameter (r_1/R) which indicates a distance from the collapse. The function Φ sign depends on the relation between x and z:

relation between x and z	Sign of Φ
0 < x < z	Φ (x) < 0
z < x < 3z	Φ (x) > 0
x > 3z	Φ (x) > 0

Clearly, there is the rupture of the function Φ(r) at the point x = 3z: the function has the infinite values of the opposite sign at different sides of this point. This rupture point position (r/R) is determined by the condition

$$9(1 - r_1^2/R^2) = 1 - r^2/R^2$$

i.e., by the known condition

$$r/R = \sqrt{9r_1^2 / R^2 - 8},$$

that leads to the additional extremum of G_{00} existence.

The initial value (at r = 0) of the pressure P is always

$$\Phi_{init}(r, r_1, R) = \frac{3 - 3\sqrt{1-(r_1/R)^2}}{3\sqrt{1-(r_1/R)^2} - 1}$$

and a boundary one (at $r = r_1$) is

$$\Phi_{bound}(r, r_1, R) = \frac{3\sqrt{1-(r_1/R)^2} - 3\sqrt{1-(r_1/R)^2}}{3\sqrt{1-(r_1/R)^2} - \sqrt{1-(r_1/R)^2}} = \frac{0}{2\sqrt{1-(r_1/R)^2}} = 0$$

References

[Novikov and Frolov, 1989] Novikov I.D., Frolov V.P. *Physics of Black Holes*. Dordrecht: Kluwer Acad., 1989.

[Tolman, 1934] R.C.Tolman. *Relativity, Thermodynamics, and Cosmology.* Oxford, Clarendon Press, 1934.

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