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Do usual gravitating bodies evaporate?

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Abstract

I analyze the Unruh's and Hawking's effects. It should note that corresponding phenomena were in fact predicted by General Relativity at the end of 1930s of the previous century. They are hold not only for a black hole but also for any gravitation source (and even electromagnetic field). Therefore, these ones also can evaporate with time.

1 Quantum Hawking's and Unruh's effects

In the middle of 1970s the works [1, 2] were published where two physically close phenomena were predicted: the black holes evaporation and a radiation that an accelerated (relative the vacuum) detector receives (effect Unruh's).

If one uses the Bogolubov's transformations to go to the accelerated observer proper time (in the Rindler coordinates) for the annihilation and creation operators [3], one can establish the temperature that the accelerated observer local thermometer measures:

$$k_B T = \frac{\hbar a}{2\pi c}$$

"Remarkably, the temperature is the same throughout the entire electromagnetic spectrum… The temperature is universal, depending only on the acceleration and on natural constants. … In all regards, the quantum vacuum of empty space appears to a single accelerated observer as thermal radiation with temperature... The thermal transmutation of the vacuum by acceleration is called the Unruh effect." [3]

Interestingly, we are able to "guess right" this formula (ignoring the numeric factor). In fact, accordingly the Wien's displacement law we have:

$$\lambda_{\max} = \frac{hc}{\gamma k_B T},$$

where γ is 0.00289776829, temperature T is in kelvins, and $\lambda_{\max}$ is in meters. Then, using the Unruh's formula we find:

$$\lambda_{\max} = \frac{hc}{\gamma k_B T} = \frac{2\pi\hbar c}{\gamma} \cdot \frac{2\pi c}{\hbar a} = \frac{4\pi^2}{\gamma} \cdot \frac{c^2}{a}$$

where we can deduce the quantity c^2 / a from the dimensionality reasons only. As one can see, the Planck constant is absent here at all.

One can consider the Hawking's effect as the Unruh's effect if an observer's acceleration is due to a motion in a black hole *gravitational* field (accordingly General Relativity). In this case we write the acceleration a in the gravitational field of mass M as $a = GM / r^2$, and express the black hole Schwarzschild's radius as $r = 2GM / c^2$, then we find the temperature of the Hawking radiation near the black hole event horizon from the Unruh's formula:

$$k_B T = \frac{\hbar c^3}{8\pi GM}$$

So, we see an accelerated observer (in my opinion) in so called thermal light state, where (like vacuum) the average electromagnetic field strength is zero, but average photons number $\bar{n} = 1/(e^{\hbar\omega/kT} - 1)$ is more than zero and average energy root-mean-square value is more than $\hbar\omega/2$.

2 Non quantum way to explain the thermal radiation

S. Hawking wrote in Abstracts of [1]:

"In the classical theory black holes can only absorb and not emit particles. However it is shown [in this article] that quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature ($\hbar\kappa/2\pi k$)≈10^{-6}(M_O/M) °K where κ is the surface gravity of the black hole. This thermal emission leads to a slow decrease in the mass of the black hole and to its eventual disappearance…" [1].

Meanwhile, as early as in 1934 R. Tolman pointed out to a temperature (and thermal radiation) existence around a gravitational bodies (not black holes) without any quantum effects:

"… the proper temperature of a fluid as measured by local observers using ordinary thermometric methods would not be constant throughout a fluid sphere which has come to thermal equilibrium, but would vary with gravitational potential, increasing with depth as we go toward the centre of the sphere. This conclusion is of course very different in character from the classical conclusion, as previously discussed …, that uniform temperature throughout is a necessary condition for thermal equilibrium. Nevertheless, from the point of view of relativity, since all forms of energy must be expected to have weight as well as mass, the conclusion that a temperature gradient is necessary to prevent the flow of heat from regions of higher to those of lower gravitational potential seems a natural and appropriate result." [4]

So, it is not necessary to associate a black hole thermal radiation with a quantum phenomenon just near the event horizon: theoretically, thermal photons can be created *at any distance* from the source, and formally we do not have to consider such the processes. The more, as "*from the point of view of relativity, since all forms of energy must be expected to have weight as well as mass"*, then a uniform temperature distribution[1] that provides the equilibrium state has to be specified for a source of any origin, for example for a usual gravitational body (not black hole!) or for a electromagnetic field source, etc.

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[1] I found that Tolman's temperature formula does not strictly coincide with Unruh's formula. Also in the recent work [5] its authors argue that the (macroscopic) Tolman temperature and the (microscopic) Unruh temperature cannot be identified, the last is due to a quantum interaction between thermometer and created particles while the vacuum temperature remains to be zero in any reference frame.

As I believe, the tunneling through the event horizon by one particle from the pair does not play any role here. We have not to associate now the evaporation with black holes: probably, it is the universal phenomenon. So, a thermal radiation carries the source energy to its environment, and the source evaporates (of course, such the effect is extremely small because the radiation temperature is negligible). Yet Hawking himself noted [1] that the relic radiation at 3 K will suppress such thermal body radiation.

3 Entropy of an arbitrary source of central field

Before the Hawking's formula derivation J. Bekenstein established the relationship for the black hole entropy S_{BH} [7]:

$$S_{BH} \approx \frac{c^3 A}{4G\hbar}$$

Here A is the surface area of the black hole event horizon.

Let us generalize such the expression for an arbitrary field source. Often one can meet the statement that it is very difficult to calculate the entropy of a gravitating mass because of self-gravitational interaction of its distant parts (in gases this effect is negligible). However, in the work [6] I proposed the solution departing from Bekenstein's point of view: *"a black hole's aspect cannot tell us precisely which type of system gave rise to it. This extra lack of "composition information" over and above that about specific microscopic configurations may be what makes black hole entropy large. A black hole stands for a large amount of missing information"*.[7]

Analogously, it is impossible to determine a matter distribution inside of a sphere of a given radius that surrounds the source, if we use only a strength measurement at a distance from its center: as it is known, such the value does not depend on the distribution! Since the gravitational field strength can be expressed trough the equivalent surface gravity then the entropy corresponding to the area of the sphere will be equal to the (dimensionless) sphere surface area. So, the analogy with a black hole seems to be clear, however, a fundamental paradox appears that was being observing by the author of the famous work [8]: if the numeric prefactor in the both formulas were the same, then the black hole entropy was much *less* than the entropy of the usual body having the same mass as its gravitational radius is much less than geometrical one!

In order to eliminate this paradox I proposed [6] to introduce some additional parameter to the factor that connects the surface area and the body's entropy. Such the parameter is the ratio ρ / ρ_{cr}, where ρ is the real body density, ρ_{cr} is the "critical" density of the collapsed body having the same mass. For example, such the ratio is near to 10^{-26} for our Earth, is near 10^{-16} for the Sun, and is close to 1 for whole the Universe (like typical black hole). It is clear, such the ration effectively increases the body entropy while the body approaches to its collapse. Also it naturally takes into account the direct correlation between the entropy and the mass inside of the sphere.

So, the proposed relationship for arbitrary body (including black holes) has to be written as:

$$S = S_{BH}\frac{\rho}{\rho_{cr}} = \frac{c^3 A \rho}{4G\hbar\rho_{cr}}$$

Let us take into account that the area A is proportional to the sphere radius square, and the density ρ is inversely proportional (at a given mass) to the sphere radius cube, so, finally, the entropy S is inversely proportional to the sphere radius, i.e. *increases while the radius decreases*. Hence, the important conclusion follows: the massive bodies attraction leads *to their total entropy increase,* i.e. such the process corresponds to the Time Arrow of the Second Law of Thermodynamics.

Some "energetic" reasons lead to similar conclusion: a free test particle attracts to the field source and minimizes the gravitation potentials difference between its current position and the source area. If such the particle orbits with a constant velocity around the source then it minimizes the algebraic sum of gravitational and kinetic energy; as result, the orbit becomes stationary.

4 Conclusions

1. The Hawking radiation that can lead to the black hole evaporation does not require especial quantum-mechanical explanation. This effect as well as Unruh's effect is due to the General Relativity accordingly to which a temperature around some gravitating body in a equilibrium state is not uniform. This temperature generates a radiation field that carries away a part of body energy of rest.
2. Generally, a central field source of an arbitrary nature (not only gravitational) has to have the similar properties. Under several conditions such the field source can "evaporate" with time.
3. I propose the generalized relationship to calculate entropy of arbitrary gravitating body (including black hole as particular case).

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