

## On the wave function's nature

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The classical macro-particle ensemble in the statistical mechanics can be described using a (matter or probability) density  $\rho$  that obeys to the Liouville equation of continuity. When one transits to the single quantum micro-particle description in order to take into account its *wave-like* features, its mass/energy (as Bohm proposed) has to be represented via a complex valued wave function  $\Psi(\vec{r}, t) = \sqrt{\rho(\vec{r}, t)} \cdot \exp[iS(\vec{r}, t)/\hbar]$  where  $S$  is a phase having the action dimension. For photons which have not the mass one can use a close approach basing on the photon *energy* notion that is the well-defined function of the *generalized* arguments – so-called electric field quadratures. A wave time period that is associated with a massive particle is near to  $10^{20}$  Hz, while a photon corresponds to a less frequency of electromagnetic field.

### 1 Schrödinger equation and Born probabilistic interpretation

As it is well known in 1926 году Schrödinger proposed the differential equation for wave function  $\Psi$  describing a quantum mechanical system. Initially he interpreted  $\Psi\Psi^*$  as a weighting function in the configuration space that describes some electro-dynamical fluctuations of the electrical charge density and concluded that one has not to search a direct interpretation for  $\Psi$  itself in the 3D space [Jammer, 1967].

A short time later Born proposed the famous probabilistic interpretation. Accordingly to it the Nature laws do not determine the event fact, but the event *probability*. Bohm interpreted the wave function modulo square integral as the particle amount, while the modulo square of each term in the wave function of a Fourier sum has to be interpreted as the state statistical frequency specified by the corresponding term.

### 2 Madelung's hydrodynamic approach and de Broglie – Bohm interpretation

In the same 1926 Madelung saw (see [Jammer, 1967]) that from the time depending Schrödinger equation one can deduce a hydrodynamic continuity equation where the density and velocity potential of a moving fluid are represented. Furthermore, Madelung showed that each eigenfunction (a solution of the wave equation) may be interpreted as some type of a steady current, although it depends on time. Because the hydrodynamic model described also another essential features of the Schrödinger equation, Madelung suggested that a possibility exists to consider the quantum atom theory from such the viewpoint. "Madelung proposed a hydrodynamic form of quantum mechanics which can be considered a precursor of Bohmian mechanics. ... Though similar to Bohmian mechanics, in quantum hydrodynamics the magnitudes of interest are the probability density  $\rho$  and the probability density current  $\vec{J} = \rho\vec{v}$ , respectively, which are related through the continuity equation." [Sanz and Miret-Artés, 2011]

David Bohm developed the Louis de Broglie idea about the wave-pilot. In 1952 he published two papers [Bohm, 1952] where he proposed to transit from the Schrödinger equation for the complex valued wave function to the system of two equations for two real quantities – *modulo* and *phase* of the wave function. Bohm considered the phase as a "hidden variable" in the von Neuman sense and this variable knowledge in principle

allows us talk about a *deterministic (not probabilistic!) trajectory of each separate quantum particle*.

A phase  $S$  in the Bohms equations turns out to be a *non-local* variable. Its dynamics depends on the specific quantum potential that in turn depends exclusively on the non-uniformity of the spatial distribution density. The quantum potential presence essentially differs the quantum description from the classical one where it is absent. The quantum potential ensures in generally the particle entanglement, i.e. the fact that the separate trajectories are not independent and cannot be described by independent wave functions. In the Bohm's model the Schrödinger equation for the wave function  $\Psi$

$$i\hbar \frac{\partial \Psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \Psi$$

of a particle having mass  $m$  in a potential field  $V$  may be transformed with the substitution

$$\Psi(\vec{r}, t) = \sqrt{\rho(\vec{r}, t)} \cdot \exp[iS(\vec{r}, t)/\hbar]$$

to the equations system:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left( \rho \frac{\nabla S}{m} \right) = 0, \quad (1)$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \quad (2)$$

where the above quantum potential  $Q$  is

$$Q = -\frac{\hbar^2}{2m} \frac{\nabla^2(\sqrt{\rho})}{\sqrt{\rho}}$$

Formally, the system of equations (1) and (2) is *completely equivalent* to the Schrödinger equation. Bohm suggested that his model describes individual particle trajectories; however, the incorrectness of the Bohm's statement directly follows from the equivalence between his model and standard paradigm. In fact, the Bohm's model describes also the *averaged* trajectories, now the averaging is not performed over *all possible* trajectories, but over the subset of them which have a fixed phase function  $S(\vec{r}, t)$ <sup>1</sup>.

Furthermore, the equation (1) is the *continuity equation* that describes a hydrodynamic ensemble, i.e. a trajectory set that was initially distributed accordingly some source density. Due to used substitution it is clear that  $|\rho| = \Psi\Psi^*$ . Thus, one can state that quantum mechanics may be considered as a statistical description with the quantity  $\rho$ . But then we come to the analogy between quantum Schrödinger equation and classical Liouville equation.

Such the analogy in a sense can be interpreted as similarity between quantum description and classical one on the base of the density distribution (if one talks about

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<sup>1</sup> Such the trajectory subsets (for which an initial state is known as well as final one) correspond to the *weak* measurement concept basing on the pre- and post-selection idea.

many particles) or probability density (when one talks, for example, about a single particle). So, the physical meaning of the wave function modulo square is essentially reduced to the energy density; note that energy is well defined for massless particles (photons) too, not only for massive ones.

Also, the quantum description is based on the complex valued wave function use, i.e., essentially uses a phase notion and then the particle wave properties. These wave properties lead inevitably to the *oscillating movement that can be attributed to a quantum object*. Such the oscillating movement can very differ for different objects, for example, in the case of a massive particle and in the case of a massless one.

### 3 Massive particle description

A quantum particle motion (contrary to a classical one) cannot be considered as *the combination of two components*: the slow one (typical for classical physics) and another one that quickly oscillates. This second component (Zitterbewegung) (for example, for electron) that satisfies to the Dirac relativistic equation was theoretically revealed by Schrödinger in 1930. The oscillation frequency ( $\sim 10^{20}$  Hz) corresponds to a particle mass of rest, i.e., is  $mc^2/\hbar$ . The oscillation phase can be considered as a “hidden variable” as well as in the Bomian model.

The fact that the oscillation frequency may be immediately expressed via the mass (therefore, via energy density  $\rho c^2$ ) just confirms the statement on the physical meaning of the *wave function modulo square*.

### 4 Photons description

The more low (less than  $10^{20}$  Hz) frequency range is specified for photons where the slow motion component is *absent*; note that the photon spatial location notion is not well defined. Also, as it is known, photons have not a mass. Meanwhile, the *energy* notion is well defined for photon ensemble and can be successfully used to interpret its wave function (see [**Sanz and Miret-Artés, 2011**]).

In fact, an electromagnetic field may be represented as an ensemble of independent *oscillators*. The each oscillator field may be in turn formally represented by a *complex valued harmonic*

$$a(x, t) = a_0 \exp[i(kx - \omega t)]$$

having angular frequency  $\omega$  and wave vector  $k$ , where  $k^2 = \omega^2 / c^2$  (here  $c$  denotes the velocity of light). One can introduce two new *real valued* variables (in the quantum optics one calls them *quadratures*):

$$q = \frac{a^* + a}{2} = a_0 \cos(kx - \omega t), \quad p = i \frac{a^* - a}{2} = a_0 \sin(kx - \omega t).$$

They are *generalized* position and momentum (respectively) for such the electromagnetic oscillator. The energy  $H$  of the oscillator is

$$H = \frac{q^2}{2} + \frac{p^2}{2}$$

But it is well known that the quadratures  $q$  and  $p$  probability distribution relative to the mean values for a coherent light beam is determined by the Wigner function

$$W(q, p) = \frac{1}{\pi} \exp \left[ -\frac{(q - q_0)^2}{2} - \frac{(p - p_0)^2}{2} \right]$$

i.e., it just corresponds to Boltzmanian energy distribution.

To conclude let us note the following. A photon that represents an electromagnetic oscillator has a *potential* energy proportional to  $(q - q_0)^2/2$  as well as *kinetic* one proportional to  $(p - p_0)^2/2$ . The sum of these two energy components is always constant and equals to  $\omega\hbar$ , however, each of them *evolves* in time as a harmonic function.

In my opinion, one neglects wrongly this fact, though it is the key to explain the wave properties of a photon behavior. Particularly, one should link a photon registration using traditional detectors with the *kinetic energy value only* at the detection moment. This energy depends on a *random* phase that well explains (using non-locality) the photon wave properties (Malus law, EPR experiments, etc.). The same fact explains an *apparent* photons absence at one of detectors in the famous interaction-free measurement experiments (see [Eltzur and Vaidman,1993]).

## References

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