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On the uncertainty relationship

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I consider three approaches to the Heisenberg's uncertainty principle: the Copenhagen interpretation, the Bohmian interpretation, and one new approach where a particle is presented as the oscillator.

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1 The Copenhagen interpretation

The uncertainty principle is one of cornestones of quantum mechanics. Firstly its proof was published by W. Heisenberg which started (see **[Jammer, 1967]**) from the simplified representations: he suggested that a particle position q is distributed around its averaged value accordingly with the Gauss error curve. Then the particle momentum p should also be distributed around its averaged value by the same manner, and the uncertainties product $\delta q \delta p$ is $\hbar/2$, i.e., the more precisely a position is determined, the less precision is specified for momentum, and vice versa. W. Heisenberg saw in this the indirect clear interpretation of the basic commutation relationship $pq - qp = \hbar/2i$.

Later Robertson showed (see **[Jammer, 1967]**) that one can formulate the general uncertainty principle for any two non-commutating operators having the commutator $i\hbar$. He proved that for a given normalized wave function and two given Ermitian operators A and B the inequality like $\Delta A \Delta B \geq \hbar/2$ can be establish.

John von Neuman reproduced this proof in his famous book **[von Neuman, 1932]**. He had in mind the *quantum* observable operators that are specified in principle by unavoidable measurement inaccuracy[1]. He stated that measurement effect to its issue just made the quantum theory (the wave function reduction) purely statistical and acausal, causes non-zero dispersion in the quantum observables ensemble distribution and excludes any possibility of the determinism recovering using any "hidden variables". Note that von Neuman pointed out to the purely *imaginairy* commutator type of two arbitrary Ermitian operators.

On the other hand, Heisenberg hi-self investigated the physical roots of his famous principle. Hi stated that measurement as such destroys the "true" picture; the more precisely a position is determined, the more a measurement device violates the "true" rmomentum value. In the textbook **[Feynman et al., 1963]** the uncertainty principle is interpreted accordingly the wave packet representations – this packet can be found in the certain range $q \pm \Delta q$ only. The uncertainty for such the packet is linked with the fact that its momentum is inversely proportional to the wavelength; however, a short wave packet is the *sum* of a number of *different* harmonics, and its *exact* wavelength cannot be well determined.

2 The Bohmian interpretation and the weak measurements

In 1952 David Bohm formulated (**[Bohm, 1952]**) that the von Neuman's statistical hypothesis is not true, and a "hidden variable" exists"[2]: it is the wave function *phase* which in principle allows us to talk about a quantum particle individual trajectory and

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[1] In the Section 3 it is shown how one can interpret the uncertainty relationship deterministically having in mind the averaging over a very small oscillation time period.

[2] Later John Bell showed that this does not contradict to the von Neuman's statement, if a "hidden variable" is not local.

velocity. Accordingly to Bohm's opinion one *can* (at least, by calculation) determine simultaneously and with arbitrary accuracy its position and momentum.

In 1988 Y. Aharonov, D. Albert, and L. Waidman proposed the idea of so-called "weak" quantum measurements. Let us measure (accordingly Heisenberg's principle) very "roughly" one of two variables. One can "overcome" this, if he performs *a number* of measurements (at a given initial and final conditions) and then *averages* the results. In this case the inaccuracy is decreased proportionally to the square root from the measurements number. Then the accuracy overcomes the theoretical limit due to the Heisenberg's relationship. It was recently performed experimentally (see **[Dixon et al., 2009]**, **[Jeff et al., 2011]**, **[Hosten and Kwiat, 2008], [Kocsis et al., 2011] [Popescu, 2009]**).

Though the amount of the publications of the Bohm's ideas and weak measurements followers increases continuously, do we can talk that the Heisenberg's relationship and quantum mechanics statistical interpretation became "obsolete"? In my opinion, this is not the case. In the both Copenhagen and Bohm's interpretations we deal with some kind of individual history particle evolution *averaging*: It is true for weak measurements by definition; for Bohmian interpretation one can see this from the formal analogy between his approach and the "probabilistic" fluid theory:

> "Consider a (classical) fluid. It consists of many different particles (e.g., atoms, ions, molecules, etc.), all the degrees of freedom being described by a set of differential coupled equations, with as many equations as degrees of freedom are involved. If one is not interested in the microscopic description of the fluid, but in a macroscopic one, equations like the Euler or Navier-Stokes ones will be rather used, which describe phenomenologically the evolution of a continuous fluid without paying any attention to the particular (microscopic) dynamics of its constituents. This is essentially the basis of classical hydrodynamics. … Thus, individual real quantum particles behave like individual point-like particles, though their distributions display wave-like behavior, in accordance with Schrödinger's equation or its Bohmian equivalents. Hence, it is clear that ensemble properties need of an ensemble description, i.e., a density distribution function, whose role is played by the probability density in quantum mechanics or, at a more elementary level, the wave function. This is in agreement with Born's statistical interpretation of quantum mechanics. … if the Bohmian equations are understood as hydrodynamic equations, the trajectories obtained from can (should) not be regarded necessarily as the trajectories pursued by real particles, but rather as the streamlines associated with the associated quantum fluid." **[Sanz and Miret-Artés, 2011]**.

See also the publication **[Sanz, 2012]** where these streamlines are identified as the lines of the mass/energy transport (that never cross in configuration space and form the "probabilistic tubes" with non-transversal boundaries); they are not the individual particle trajectories! Thus, the Bohm's model that is mathematically equivalent to the "diffusional" Shrödinger's equation cannot (and does not) describe any chaotic ("Brownian") motion of each individual particle; it gives averaged (collective) motion description only.

3 The uncertainty relationship and a particle model as the elementary oscillator

The concept of "quantum particle" is usually associated with its image as a material point that could be specified by *real* values of its position and momentum. However, this concept was destroyed by quantum mechanics founder – Heinsenberg him-self. He introduced (see **[Jammer, 1967]**) a quantum particle position and momentum as infinite serie of *complex* valued harmonics. He also introduced several limitations for these quantities multiplication rules (identical with the rules for matrix) from what he deduced all the rest. Do we can found some clear physical interpretation of such the mathematical scheme based on the classical theory?

In my book **[Shulman, 2004]** I proposed consider a quantum particle model as complex valued oscillator. From the *mathematical* viewpoint this completely corresponds with the Heisenberg's approach. From the *physical* viewpoint this explains the representation complication about particle position and trajectory, about physical quantities commutation and their simultaneous measurements, about proper angular momentum, about two particle types (fermions and bozons).

In this case we can imagine the motion description as two components combination: slow (typical for classical physics) and fast (oscillating). Such second particle motion component ("Zitterbewegung") (for example, electron) that obeys to the Dirac relavistic equation was theoretically found by Shrödinger in 1930. The oscillation frequency corresponds to the transitions from particle to anti-particle (near $2mc^2/\hbar \approx 10^{20}$ Hz). The oscillation phase like the Bohm's model can be considered as the hidden non-local variable. The quantum inaccuracy of measurement is just connected with this oscillation.

Let us introduce the instant *action*:

$$s(t)=q(t)p(t)=q_0 \sin \omega t \cdot p_0 \sin (\omega t+\varphi)]$$

This expression can be written as the sum of two components

$$s(t)=q(t)p(t)=(q_0p_0) [\cos \varphi (\sin \omega t \sin \omega t) + \sin \varphi (\sin \omega t \cos \omega t)]$$

The first term in the square brackets is proportional to cos φ and represents so-called *active* component. It does not contain a phase shift between two harmonic factors. This component describes the irreversible energy lost per period in the oscillator, it may be calculated by direct integrating over period:

$$S_a=(q_0p_0) \cos \varphi$$

The second term in the square brackets is proportional to sin φ and represents so-called *reactive* component. It contains the phase shift between two harmonic factors equal to π/2. This component describes the purely oscillating energy, its averaged value is of course equal to zero.

When φ=0, then all the energy of the oscillator should be dissipated during the period; when φ= π/2, then the oscillator total energy over the period is conserved.

It is easily to show that the *total* variance of two components relative to the mean value S_a is

$$D = (q_0p_0)^2 (\cos^2\varphi + \sin^2\varphi)/8=(q_0p_0)^2/8$$

If we transit from the amplitudes q_0 and p_0 to the effective values q_{eff} and p_{eff}, then we have the Heisenberg's relationship:

$$D = (q_{eff}p_{eff})^2/4$$

where we have to replace the *classic* action ($q_{eff}p_{eff}$) prefactor by the *quantum* constante ħ.

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