On quantum superposition of particle's trajectories

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One usually describes a double-slit experiment like classical interference. At this one introduces a wave function (complex) amplitude instead of a "real" wave amplitude, and the square of the wave function amplitude modulo gives the probability value (the Born rule). R. Feynman considered this as the main quantum mechanics mystery. Here we come to the same viewpoint by using standard rule of the *consistent* alternatives addition of the probability theory, i.e., while one physically admits a possibility for both particles to travel through two splits simultaneously (not through one of them only). By such a way we explain a gradual revealing of an interference pattern in a number of experiments with single particle exclusively by the statistical features of individual experiments, not by some ensemble collective properties (e.g., "entanglement in time"). We also discuss the similar situation with the quantum walk alternative trajectories of Cs atoms in experiments where the Leggett – Garg inequalities were tested.

1. Double-slit experiment description

As it is known, a quantum system and process can be in the superposition state. For example, one can suggest that that the particle trajectories in a double-slit experiment may be superposed.

R. Feynman described the double-slit experiment with single electron in his famous manual on Physics [1]. If particle detectors are positioned at the slits, showing through which slit a photon goes ("which way information" is present), the interference pattern is absent, and we have

$$A^2=A_1^2 + A_2^2, \qquad (1a)$$

where the square of the total wave amplitude A is equals to the sum of two partial (harmonic) wave amplitudes A_1 and A_2. However, when one does not know through which slit a particle goes ("which way information" is absent), than the interference pattern can be revealed like the case of two coherent classical waves:

$$A^2=A_1^2 + A_2^2 + 2\, A_1\, A_2 \cos \Delta\varphi, \qquad (1b)$$

i.e., the interference term with the phase difference $\Delta\varphi$ between two wave propagation lines in the observation point is added.

The formulae (1a, 1b) contain the particle current *amplitudes*. So, one implicitly considers some *collective* particles movement that aren't separated by sufficient time intervals. However, when two *single* particles separated by a large time intervals travel through the slits one after another we have to talk about the *probability* for a single particle to travel by one way or another, not about the current amplitude. Meanwhile, a transition from the current amplitude to the single particle probability does not present a difficult problem – this probability just has to be simply proportional to the corresponding amplitude square.

R. Feynman uses the notion of *complex probability amplitude a* instead the typical harmonic source amplitude [1]. Such the probability amplitude multiplied by a phase factor is equal to the square root from the event probability P in the perfect experiment: $|a|^2$ = P. If an event can be realized by a number of ways, then we have to add the amplitudes for each possible way, and the interference appears: $a = a_1 + a_2$, and P = $|a_1 + a_2|^2$; to calculate the interference terms we have to multiply one amplitude by the conjugate to another. However, if the experiment's arrangement allows us to be informed about of *which way was really realized*, then the total probability is simply the sum of the partial probabilities, the interference is absent: $P = |a_1|^2 + |a_2|^2$.

2. The formalism to describe a superposition

Let us try to transit to the probability more straightforward and clear. It is useful to apply the total probability formula $P(A_1 \cup A_2)$ for two *joint alternatives* [2], i.e., to admit that one of them does not excludes the other one. In such the case the total probability can be written as

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2). \quad (2)$$

where:

$P(A_1) \sim A_1^2$ is the probability of the alternative A_1 only;
$P(A_2) \sim A_2^2$ is the probability of the alternative A_2 only;
$P(A_1 \cap A_2)$ is the probability of the simultaneous alternative A_1 and A_2 realization.

In order to obtain the similarity between (2) and (1b) let us now write[1]:

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) + 2K_0. \quad (3)$$

Here we introduced the correlation momentum K_0 between alternatives A_1 and A_2:

$$K_0 = -P(A_1 \cap A_2)/2. \quad (4)$$

As total probability has to be equal to the unit

$$P(A_1 \cup A_2) = 1, \quad (5)$$

we find from (3):

$$K_0 = -[P(A_1) + P(A_2) - 1]/2. \quad (6)$$

From (1b) and (3) we can conclude that in the double-slit experiment we obtain

$$K_0 \sim (-A_1 A_2 \cos \Delta\varphi). \quad (7)$$

This means that the correlation momentum K_0 is proportional to the correlation function $\cos \Delta\varphi$.

3. Is there an entanglement in time between double-slit trajectories?

Do we have to say that the particle trajectories should be "entangled in time"? For example, the work [3] describes the behavior of the single-electron beam in a biprism whose features are similar to a double-slit screen. In the experiments it was possible to accumulate gradually and record the results of the electrons travelling.

As single spot number on the screen increased (up to 200 000), an interference pattern gradually appeared. So, we come to the dilemma:

- either we meet some "collective" behavior of electrons where (at least) the actual trajectories "know" the previous distribution of the single spots on the screen; if so, the duality particle-wave concept has a sense at a collective description only;

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[1] It is useful to remember that the total dispersion of two observables is [2]: $D(A_1 \cup A_2) = D(A_1) + D(A_2) + 2K_0$

- or a single electron interferes with it-self (more precisely, two possible electron motion trajectories interfere), i.e., in each experiment involving single electron the probability spot distribution is described by (1b), not by (1a); in such the case the duality particle-wave concept has a sense just at a single description.

As we believe, it is not useful to discuss first scenario and describe the electron ensemble only. Similarly, it not useful to state that a single particle having some thermal speed “mystically knows” the value of the whole ensemble average temperature. The statistics appears independently due to the large numbers law, as well as macroscopic interference pattern too.

4. Another example of the trajectory superposition

In 2015 the scientific group from Germany and United Kingdom performed the experiment [4] in which the quantum walk of a massive quantum particle (Cs atoms) was investigated. The results rigorously exclude (i.e., falsify) any explanation of quantum transport based on classical, well-defined trajectories. Contrary, they confirm the atom displacement as a witness of quantum alternative trajectories superposition. This work was performed in order to test the Leggett – Garg temporal inequality [5] that distinguishes classical (well defined, they aren’t in a superposition) states from quantum ones (being in a superposition).

Let us now make two remarks. First, in their original paper [5] Leggett and Garg investigated namely states, not trajectories; however, this fact was not explicitly noted in [4]. Second, as we believe, just the fact of the correlations existing may form the simpler superposition witness than the Leggett – Garg inequality satisfaction. Indeed, in our work [6] we demonstrated that the inequality violation depends exclusively on the three correlation function values between dichotomous observables. This forces the Leggett’s original idea that namely correlative links distinguish (inconsistent in principle) classical alternatives from quantum ones (which may be consistent). Because of that, in our opinion, just the fact of non-zero correlation function is the witness of the alternatives non-classicality. E.g., for the set of alternative trajectories from [4] one could compare the probabilities P_1, P_2, …, P_n (of the Cs atom travelling between initial and final points) corresponding to known way with the probability P_0 corresponding to unknown way. If the probability sum P_1, P_2, …, P_n is not equal to P_0, then this means the *amplitudes* addition, not the *probabilities* one, i.e. the trajectories existing is confirmed.

Is the correlation existing sufficient condition of the physical alternatives non-classicality? Generally, the answer is negative, because in classical physical systems some mutual links or feedbacks leading to correlativity between alternatives may appear. However, in the Legget’s approach it may be sufficiently enough, because we talk about the well defined states that do not depend on other possible states (macro-realism). On the other hand, although it seems be surprising, Leggett-Garg inequalities do not form an optimal tight boundary for macrorealistic theories (like Bell’s inequalities do for local realism), they unnecessarily limit the parameter space in which potential violations of macroscopic realism can be found [7].

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