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BELL'S THEOREM AND LOCAL INDETERMINISM

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As it is well known, the different deterministic models cannot explain the results of EPR-experiments and the Quantum Mechanics predictions. In addition to these models a local *indeterministic* one due to the Malus law is considered in the paper. It is shown that this model is also unable to explain the remote EPR-correlations.

Introduction

As it is known, a pair of coherent photon flies away in the opposite directions from a common space point when one tests a Bell's inequality violation in the EPR-experiment. There are two polarizers at each side of the setup that are separated by a spacelike distance. The coincidence counts of photons passing through both polarizers should be computed in the experiment.

Note, Quantum Mechanics (QM) predictions for the coincidence counts and the correlation function dependence on an angle difference between polarizer orientations was surely confirmed in such type of experiments (see **[Aspect, 2000]**, **[Weihs et al., 1998]**). The numerous attempts to explain the experiment results using some hidden variables theories (LHV) (i.e. any common determined cause leading to EPR-correlations) were unsuccessful. Further, the Bell's theorem theoretically showed the groundlessness of all the Local Realism Models basing on a local deterministic measurements. Last years the experiments were performed that disproved the non-local hidden local variable (NHV) theories (see **[Branciard et al., 2007]**, **[Paterek et al., 2007]**). All this leads to any "superluminal" influence between two remote events and some collision between the Relativity and QM (see **[Aspect, 2000]**, **[Gizin, 2005]**).

However, the Bell's theorem condition is sufficient but not necessary one to violate the Bell's inequality **[Khrennikov, 2007]**. It means this one can be violated in a different case too. Indeed, Bell applied the deterministic measurement model, when a hidden variables strongly determine the measurement outcome. But, as I know, until now nobody considered any Local *Indetrministic* Measurement Model (LIMM), when a hidden variables determine the measurement outcome with any probability, although the indeterminism just presents a corner stone of QM. I believe, the QM's non-locality cannot be considered as complete one without examination of such models.

Experimental testing of EPR-correlations

In practice one uses the coincidence counts N_{++} , N_{+-}, N_{--} , and N_{-+} in the EPR-experiments to test Bell's inequality. For example, N_{++} is the amount of events when each of two coherent photons passed through its polarizer. Analogously, N_{+-} is the amount of events when one of photons passed through the first polarizer, and another photon did not pass through the second polarizer, etc. The total amount N of events is the sum of four coincidence counts, i.e. т.е. the total amount of the accounted coherent photon pairs. Further, the correlation function may be determined as:

$$K = (N_{++} + N_{--} - N_{+-} - N_{-+})/N = P_{++} + P_{--} - P_{+-} - P_{-+}$$

where P are respectively normalized event probabilities.

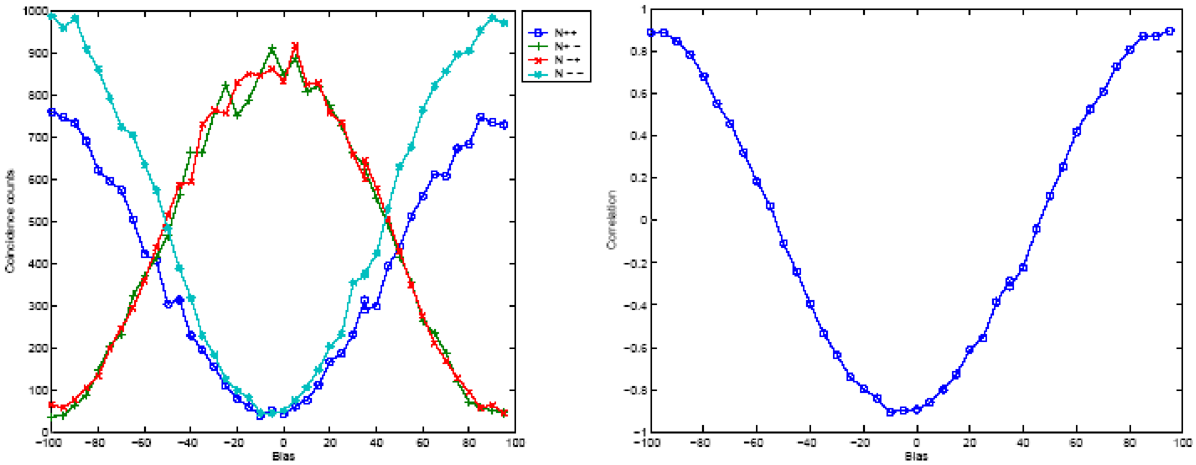


Figure 1 **[Adenier and Khrennikov, 2006]**.
At the left: the coincidence counts depending on a angle difference between polarizer orientations **[Weihs et al., 1998, Innsbruck]**. At the right: the correlation function

The typical experimental data **[Weihs et al., 1998]** are plotted on the Fig. 1 (see **[Adenier and Khrennikov, 2006]**). Although several real data are not sometimes follow expected ones, the conclusions usually confirm the QM predictions. Note, the correlation function amplitude as well as minimal and maximal absolute values of the coincidence counts are less than theoretically expected ones, however the differences are small and often can be explained by an experimental details.

Local Indeterminism Measurement Model (LIMM)

Such kind of measurement model in the EPR-experiments one can build using the Malus law: if an angle between a light beam polarization and the polarizer optical axis is $\boldsymbol{\theta}$, then the intensity part proportional to $\cos^2 \boldsymbol{\theta}$ will pass only through the polarizer. So, a real polarizer allows measure the passing radiation intensity. If the radiation is monochromatic (each photon has the same energy), then we really measure a photon amount passing through the polarizer per a time unit.

However, it is true when the photon amount is much more than one at the same time moment (a classical measurement). But when the single photon passes through the polarizer we have the independent quantum measurement and the QM probabilistic language should be used to describe the test.

Let λ being a “true” photon polarization (a hidden variable), $\boldsymbol{\theta}_{\mathbf{I}}$ being the first polarizer setting, $\boldsymbol{\theta}_{\mathbf{II}}$ being the second polarizer setting. Using the Malus law one can determine the probabilities P_{1+} and P_{1-} for a photon to pass or not to pass through the first polarizer at a given $x = \lambda - \boldsymbol{\theta}_{\mathbf{I}}$:

$$P_{1+} = \cos^2 x, \qquad P_{1-} = \sin^2 x$$

Analogously, we can write for second polarizer

$$P_{2+} = \cos^2(x+ \theta), \qquad P_{2-} = \sin^2(x+ \theta)$$

where $\theta = \boldsymbol{\theta}_{\mathbf{I}} - \boldsymbol{\theta}_{\mathbf{II}}$, because the photons are coherent and $\lambda - \boldsymbol{\theta}_{\mathbf{II}} = (\lambda - \boldsymbol{\theta}_{\mathbf{I}}) + (\boldsymbol{\theta}_{\mathbf{I}} - \boldsymbol{\theta}_{\mathbf{II}}) = x+\theta$.

If we consider the single (+) and (–) events as independent ones we will find for two polarizer joint outcomes by multiplying:

$$P_{++}(x, \theta) = \cos^2 x \cos^2(x+\theta), \qquad P_{+-}(x, \theta) = \cos^2 x \sin^2(x+\theta),$$

$$P_{--}(x, \theta) = \sin^2 x \sin^2(x+\theta), \qquad P_{-+}(x, \theta) = \sin^2 x \cos^2(x+\theta)$$

It is easy to see that the sum of these four quantities is equal to 1, so the normalization condition is fulfilled. This normalizing will remain correct after averaging (integration) over all the possible x values (from 0 up to 2π), this operation will exclude a dependence on x:

$$P_{++}(\theta) = P_{--}(\theta) = \tfrac{1}{4}\,(1 + \tfrac{1}{2}\cos 2\theta), \qquad P_{+-}(\theta) = P_{-+}(\theta) = \tfrac{1}{4}\,(1 - \tfrac{1}{2}\cos 2\theta),$$

$$K = P_{++} + P_{--} - P_{+-} - P_{-+} = \tfrac{1}{2}\cos 2\theta$$

Let us compare now this result with QM prediction:

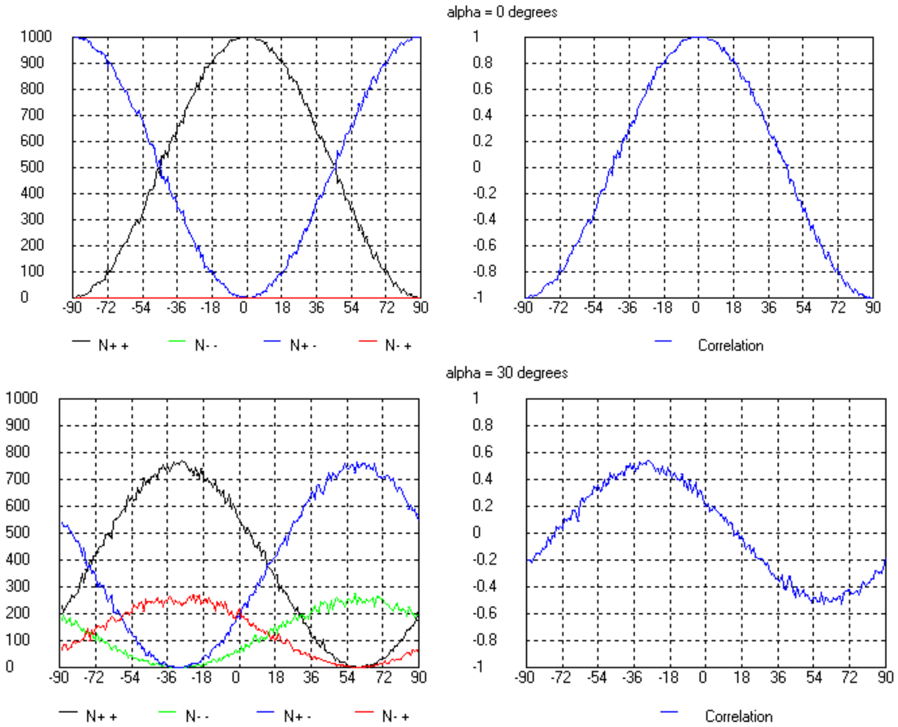
$$P^{QM}_{++}(\theta) = P^{QM}_{--}(\theta) = \tfrac{1}{4}\,(1 + \cos 2\theta), \qquad P^{QM}_{+-}(\theta) = P^{QM}_{-+}(\theta) = \tfrac{1}{4}\,(1 - \cos 2\theta),$$

$$K^{QM} = P^{QM}_{++} + P^{QM}_{--} - P^{QM}_{+-} - P^{QM}_{-+} = \cos 2\theta$$

Note two important and interconnected differences: 1) LIMM predicts K and second term of P values (depending on 2θ) two times less than QM; 2) LIMM predicts the non-zero positive minimum for P values (however, QM predicts zero for them).

Computer simulation

It is not difficult to study the proposed local indeterministic model analytically. However, a computer simulation of such EPR-test seems to be more commode and visual.



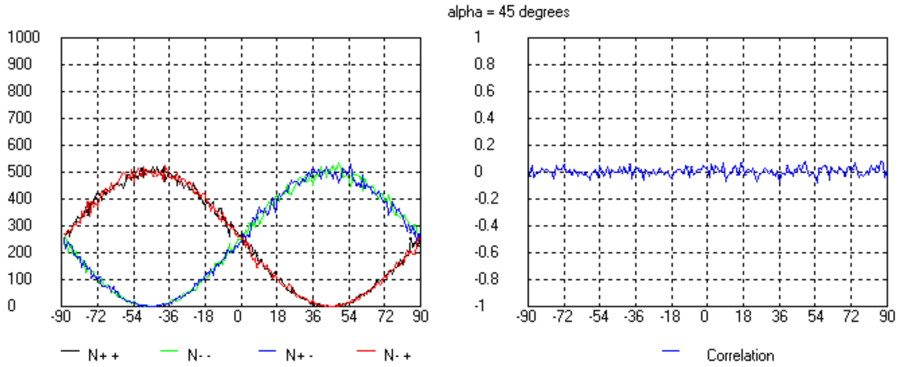


Figure 2.
Simulation of the EPR-experiment at fixed angles between photon polarization and polarizers axis (coincidence counts on the ordinate axis, the angle in degrees on the argument axis).

It consists in the event accidental selection like a photon “passes” or not through the polarizer. If we know the λ and $\boldsymbol{\theta}_{\mathbf{I}}$ values, then “the outcome” (1 or 0) for the first polarizer is determined using random number generator: if a “registration” probability $\cos^2(\lambda - \boldsymbol{\theta}_{\mathbf{I}})$ is less than a random number, then “outcome” should be set to zero (photon did not pass through polarizer), else it should be set to the unit. Analogous rule acts for the second polarizer, however the second probability should be equal to $\cos^2(\lambda - \boldsymbol{\theta}_{\mathbf{II}})$. After each step the corresponding coincidence counts are incremented.

Note that the computer model (unlike a real experiment) allows in fact control the parameter λ, i.e. a “true” photon polarization (as it really existed before measurement!). The fig. 2 shows the observables behavior at different $\alpha = \lambda - \theta_I$ in the EPR-experiments. When the *averaging over all the possible values λ is absent* (i.e. the angle between photon polarization and polarizer axis is fixed) the coincidence counts N_{++} and N_{--} (as well as N_{+-} and N_{-+}) are not in general equal one to another, and the function correlation amplitude can vary from 0 up to 1.

Contrary, when one *averages over all the possible values λ*, the new effect appears as Fig.3 shows: *the count plots lift up, so their minimal value become positive*. It is easy to show that analytically: for instance, we have at $\theta = \pi/2$

$$P_{++}(\pi/2) = P_{--}(\pi/2) = \tfrac{1}{4}\,(1 + \tfrac{1}{2}\cos\pi) = 1/8 \;>\; 0$$

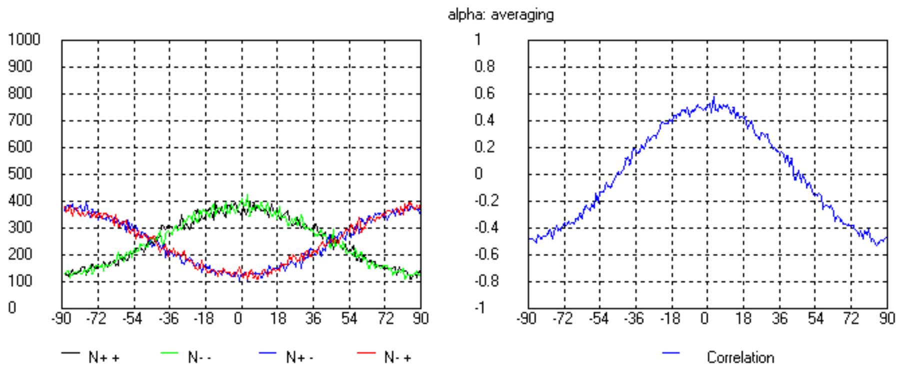


Figure 3.
The EPR-experiment simulation with uniform averaging of the difference angle between photon polarization and polarizer axis orientation.

Also, the correlation function depending on the angle 2θ has the amplitude equal to 0.5. In general, this amplitude can become more if the distribution ρ(λ) is non-uniform, however, in this case the counts become very divergent (see Fig. 4) contrary to the experimental results .

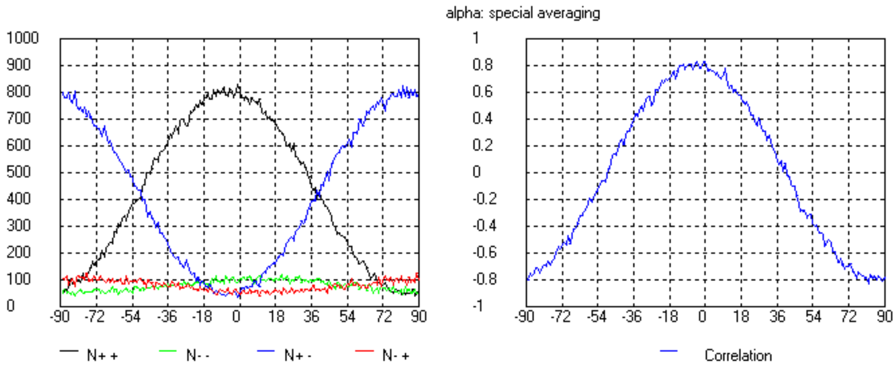


Figure 4.
The EPR-experiment simulation with non-uniform averaging of the difference angle between photon polarization and polarizer axis orientation.

Conclusion

One can see that the experimental correlation function amplitude (90%) is near to the QM prediction (100%) and is not in agreement with the LIMM prediction (50%). The more, a Bell's inequality violation is impossible at the 50%-amplitude value of the correlation function.

The non-zero minimal coincidence count value contradicts to QM as well qualitatively as quantitatively. Indeed, QM predicts that the coincidence (or not) fact depends only on an angle difference ($\boldsymbol{\theta}_{\mathrm{I}} - \boldsymbol{\theta}_{\mathrm{II}}$) between two polarizer orientations. Particularly, if this difference is equal to $\pi/2$, then the coincidence is *completely impossible*. Contrary, in the LIMM corresponding to the Malus law some coincidence is possible even if the polarizer orientations are orthogonal but a difference between λ and $\boldsymbol{\theta}_{\mathrm{I}}$ (or $\boldsymbol{\theta}_{\mathrm{II}}$) is non-zero (see Fig. 5). It is just the cause of the plots lifting up on the Fig.3 (at the left) when one averages over all the values λ.

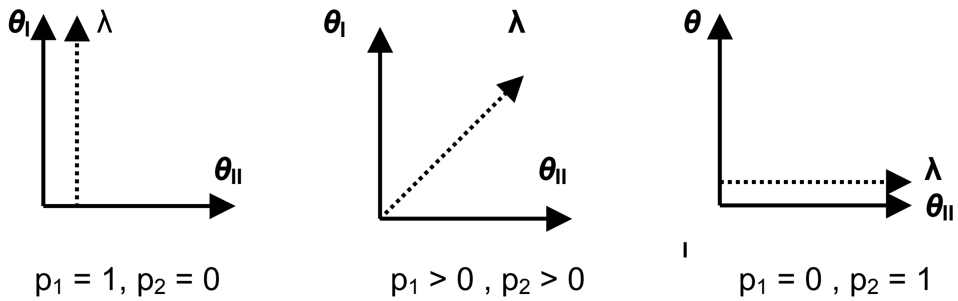


Figure 5.
The Local Indeterminism Measurement Model predictions when the polarizers axis are orthogonal.

This conclusion is enough general. If even LIMM was based on a law different from the Malus one, the averaging over λ followed a non-zero contribution to the minimal coincidence count values. Note, it is false for *deterministic* models: for example, in the "naive" model from the paper **[Aspect, 2000]** the photon passing through a polarizer is determined by the condition, is the absolute angle difference between a photon polarization and the polarizer orientation less (or not) than $\pi/4$; for such model we get the plots presented on the Fig. 6.

So, I belive that Bell's theorem treating only "realistic" measurements can be expanded up to *Generalized Bell' Statement*: ***all the local*** *(as well indeterministic as deterministic)* ***measurement models*** *are unable to describe the EPR-experiment results corresponding to QM predictions.*

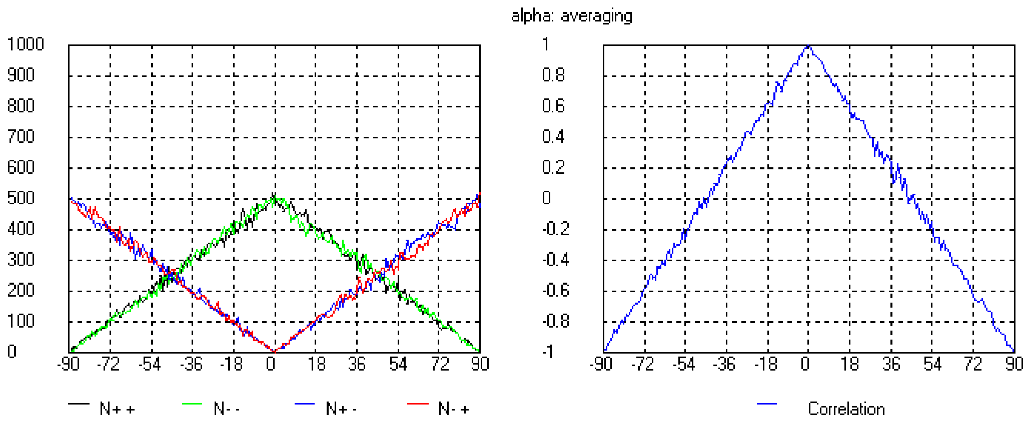


Figure 6.
The EPR-experiment simulation (deterministic "naive" model) with uniform averaging of the difference angle between photon polarization and polarizer axis orientation.

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