Inequalities of Bell and Leggett, and quantum superposition

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The inequalities of Bell were deduced from two premises: (a) real existence of concrete measured values *before the measurement*, and (b) locality, i.e. the absence of any inference between two spatially separated measuring devices that detect the quantum particles.

These inequalities violation shows that one of the premises or both of them are not satisfied. The attempt to reveal a true cause of such violation is in area of invention of new experiments that should separate the non-locality and classical reality. This fact recognition was verified in [1], and some concrete experiment proposal was given in [2] where the absence of a priory photon number in a field *before* detection was established. The locality hypothesis was not used in [2], although one could invent some absurd realm where photons jumped from one detection channel to another through an opaque wall or there existed some spooky connection of an unknown origin.

The further investigations in this frame are connected with the paper of Leggett and Garg [3], who proposed the temporal inequality in which the spatially separated measuring devices are absent, so a non-locality hypothesis isn't needed; additionally, in the paper [4] Leggett proposed another (spatial) inequality for two entangled quanta whose violation allows us to exclude one type of non-local connection (*a measuring device interconnection*, e.g., a measurement result dependence on the mutual analyzers orientation); the experiment was performed (see [5]). However, these papers cannot exclude another type of non-locality, when a *measurement result* of one detector depends on a *measurement result* of another one. Nevertheless, the existed theoretical and experimental data, as one believe, allow us to surely state the absence of well determined observable values before their detection, i.e., a quantum superposition real existence (e.g., when Schrödinger's cat is simultaneously alive and dead).

1. Introduction

The inequalities of Bell were deduced from two premises: (a) real existence of some measured values *before the measurement*, and (b) locality, i.e. the absence of any inference between two spatially separated measuring devices that detect the quantum particles. Let us remember that standard inequality of Horn, Clauser, Shimony, and Holt has the form [6]:

$$|E(a,b) - E(a,b') + E(a',b) + E(a',b')| \leq 2, \qquad (1)$$

where a and a' are the analyzer settings on the side A, b and b' are the analyzer settings on the side B, four combination are tested in four separate experiments. The terms $E(a, b)$ etc. present quantum correlations for particle pairs where quantum correlation is defined as the product expectation of corresponding experimental "results" (+1 or -1), i.e., the statistical average of $A(a)\cdot B(b)$.

Quantum mechanics predicts that $E(a,b)=\cos\varphi$, where φ is doubled difference between analyzer angles for settings a and b. Generally, the angle differences of three terms may not be equal one to another; the fourth one is the simple function of first ones. It follows from the symmetry reasons that a maximal violation of the inequality (1) will satisfied, if we take all three angle differences in (1) being the same ones. If one denotes their value as φ, then he can easily derive the resulting inequality from (1):

$$| 3 \cos\varphi - \cos 3\varphi | \leq 2. \qquad (2)$$

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The figure 1 shows that the left side of (2) plot overcomes the horizontal limit 2 (the right side of inequality) at several values of φ; such the fact differs quantum situation from the classical one.

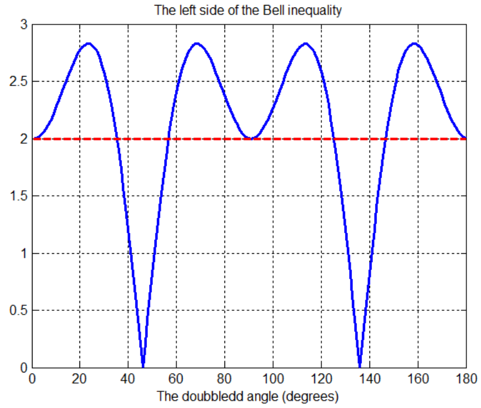

Figure 1 (color online). The CHSH-inequality at several values of the doubled analyzer angle difference

The inequality (2) violation shows that one of the premises (a) and (b) or both of them are not satisfied. The attempt to reveal a true cause of such violation is in area of invention of new experiments that should separate the non-locality and classical reality. This fact recognition was verified in [1], and some concrete experiment proposal was given in [2] where the absence of a priory photon number in a field *before* detection was established. The locality hypothesis was not used in [2], although one could invent some absurd realm where photons jumped from one detection channel to another through an opaque wall or there existed some spooky connection of an unknown origin.

Meanwhile, one can meet other viewpoints in a literature. For example, the known investigator of the quantum non-locality problem A. Khrennikov (the chairman of a number of international workshops) in [7] states that physicists simply incorrectly use the modern probability theory, they implicitly consider a "non-Kolmogorovian" models. He writes: *"Non-Euclidean local effects might be also imagined as nonlocal Euclidean effects. In probability theory the Kolmogorov probability model is an analogue of the Euclidean geometry. In this review we present the viewpoint that violation of Kolmogorovness might be interpreted as "death or reality" or non-locality. However, this is death of only Kolmogorovian reality. Kolmogorov nonlocalty might be in fact simply non-Kolmogorov locality*."

Further, the author of [7] says: "*In classical probability theory researchers never try to put statistical data collected on the basis of different sampling experiments into one single probability space. However, in quantum mechanics we (at least Bell and his adherents) try to do this. From the point of view of "the probabilistic opposition" to the conventional interpretation of violation of Bell's inequality, the crucial problem of Bell's considerations was placing statistical data collected in a few totally different experiments (corresponding to different setting of polarization beam splitters) in one probabilistic inequality*". In order to base his statement the author of [7] gives an example (from [8]) including three random discrete variables taking only two values (+1 or -1). Suppose that these variables as well as their pairs can be measured and hence joint probabilities for *pairs* are well defined. However, generally it is impossible[3] to construct the joint probability distribution for any *triple* of random variables because several of these ones can become *negative*! This fact contradicts to the Bell inequality for correlations. The author writes: "*In accordance with Boole we consider Bell's inequality just as a necessary condition for probabilistic compatibility.* "

[3] This general result, as is noted in [7], was established for hundred years ago by Boole (who invented Boolean algebras).

Thus, A. Khrennikov explains a violation of the Bell inequality by some incorrectness in probability theory usage (and negative probability appearance), not by locality hypothesis. Meanwhile, in the paper [2] the Bell inequality was deduced like the CHSH inequality directly from assumption of non-negativity of involved probabilities. However, the author of [2] concludes that the locality assumption was implicitly accounted because the first observer outcome was considered being independent on the measuring device of second observer, i.e. its settings.

2. The spatial Leggett's inequality

In 2003 Leggett proposed [4] the inequality for spatially separated objects; its violation is able to exclude a type of non-locality when a *non-local connection* exists between the measuring devices (say, between analyzer mutual orientations). However, this work is unable to exclude another non-locality type, namely the *measurement outcome dependence* of one detector on another. Nevertheless, the existed theoretical and experimental data, as one believe, allow us to surely state the absence of well determined observable values before their detection, i.e., a quantum superposition real existence (e.g., when Schrödinger's cat is simultaneously alive and dead).

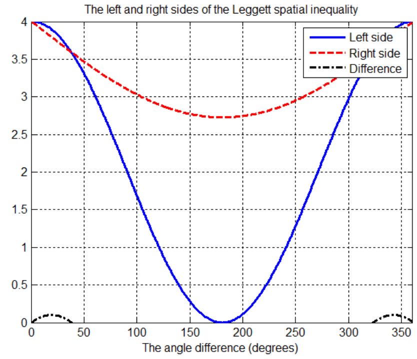

Figure 2 (color online). The spatial Leggett inequality violation at several values of the angle difference between the polarizer orientations.

In the work [5] the Leggett spatial inequality was adopted in order to test it in the experiment that was described in the same paper. The authors of [5] performed the generalized EPR-experiment: one observer (Alice) selects from two analyzer settings $\vec{a}_1$ and $\vec{a}_2$ while another observer (Bob) selects from three analyzer settings $\vec{b}_1$, $\vec{b}_2$ and $\vec{b}_3=\vec{a}_2$. They achieve the following generalized spatial Leggett inequality:

$$|E_{11}(\varphi)+E_{23}(0)|+|E_{22}(\varphi)+E_{23}(0)|\le 4-\frac{4}{\pi}|\sin\frac{\varphi}{2}|, \tag{3}$$

where $E_{kl}(\varphi)$ is a uniform average of all correlation functions, defined in the plane of $\{\vec{a}_k, \vec{b}_l\}$, with the same relative angle φ. For the inequality to be applied, vectors $\vec{a}_1$ and $\vec{b}_1$ necessarily have to lie in a plane orthogonal to the one defined by $\vec{a}_2$ and $\vec{b}_2$ on the Poincaret sphere. As it is noted in [5], the quantum correlation function for measurements $\vec{a}_k$ and $\vec{b}_l$ depends only on the angle differences between these vectors therefore $E_{kl}=-\cos\varphi$, so the quantum prediction for (3) became

$$|2(\cos\varphi+1)|\le 4-\frac{4}{\pi}|\sin\frac{\varphi}{2}| \quad . \tag{4}$$

The plots of the inequality (4) left and right sides are shown in the figure 2 as well as difference between them. The inequality maximal violation occurs at φ=18.8º as experiment confirmed.

3. The temporal inequality of Leggett - Garg

The switch from spatially separated measurements to a measurement on the same object but performed at different times is seemed to be very surprising. In the paper [3] Leggett and Garg proposed the *temporal* inequality concept where the spatially separated observations are absent and therefore locality condition is not needed.

The authors of the work [3] considered (as a qubit) a rf QUID presenting a ring of superconducting material interrupted with one or more nonlinear elements known as Josephson junctions. Such circuit defines two possible states of magnetic flux threading the ring. When a measurement is made, the qubit is found in one of the two possible states with a probability that oscillates in time (so-called "Rabi oscillations"). Authors of [3] assign to an observable Q the value +1 for the first possible state and -1 for the second one. They propose to repeat three times the dichotomous observable measurements Q, then one obtains the outcomes Q_1, Q_2, and Q_3.

Then following inequality holds[4]:

$$| Q_1Q_2 + Q_2Q_3) | \le | 1 + Q_1 \cdot Q_3 | \qquad (5)$$

Further, such triple of measurements has to be performed many times in order to calculate the expectation of products Q_1Q_2, Q_2Q_3, and Q_1Q_3, and find the corresponding correlation functions k_{12}, k_{23} и k_{13}. Thus, the temporal Leggett – Garg inequality appears that will hold if the flux is in a well defined state (not in their superposition) between measurements:

$$| k_{12} + k_{23}) | \le | 1 + k_{13} | \qquad (6)$$

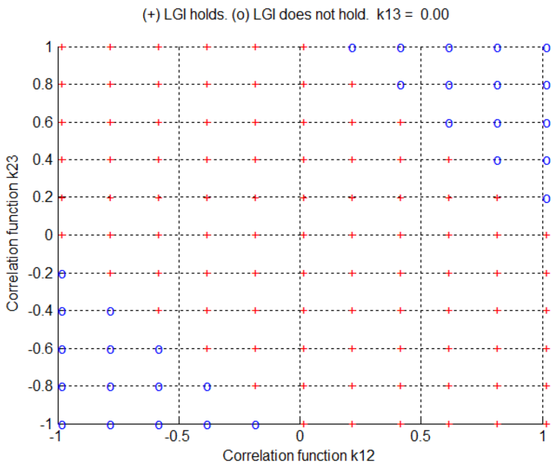

Figure 3. The area where the inequality (6) holds.

The red symbols ("+") denote the points where the inequality (6) holds (i.e., the left side is less than or equal to the right side), and the blue symbols ("o") denote the points where inequality (6) is not satisfied (i.e., the left side is more than the right side). The horizontal axis presents values of k_{12} (from -1 up to +1), the vertical axis demonstrates the values of k_{23} (from -1 up to +1).

The correlation functions k_{12}, k_{23} и k_{13} varying (in the range from -1 up to +1) show that in this case their values also determine whether the basic inequality (4) will be satisfied. The area

[4] *Proof*: 1) If $Q_1 \cdot Q_3 = -1$, then $Q_1 + Q_3 = 0$, $(1 + Q_1 \cdot Q_3) = 0$, i.e. $0 \le 0 \le 0$.
2) If $Q_1 \cdot Q_3 = 1$, then $Q_1 + Q_3 = \pm 2$, $(1 + Q_1 \cdot Q_3) = 2$, i.e. $-2 \le (Q_1Q_2 + Q_2Q_3) \le 2$.
But the value $|Q_1Q_2 + Q_2Q_3|$ cannot be more than 2, so the inequality is satisfied. ■

where the inequality holds is presented in coordinates k_{12}, k_{23} (and at some fixed value k_{13}) by a diagonal region that continues from the left upper corner up to the right lower one. As k_{13} varies from -1 up to +1, this region expands relative to this thin diagonal line and finally (at k_{13}=1) fills all the diagram (see, for example, figure 3 at $k_{13} = 0$).

Yet we have plotted the Figure 4 that shows increasing a fraction of points for which the inequality (6) holds and maximal violation of (6) decreasing as correlation function $k_{12} = k_{23} = k_{13}$ increases from -1 up to +1.

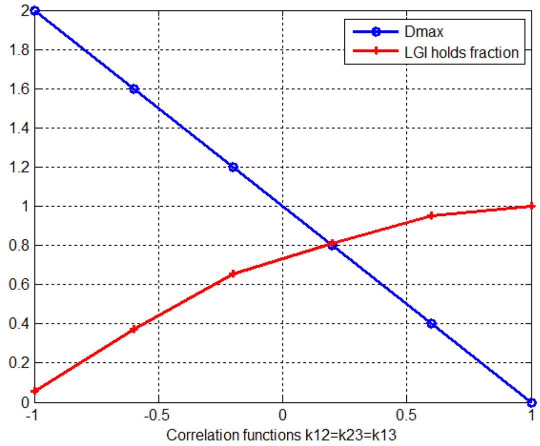

Figure 4 (color online). Dependence of the maximal inequality (6) violation and fraction of the points amount for which LGI holds vs correlation function $k_{12} = k_{23} = k_{13}$.

4. Fundamental quantum inequalities and the correlativity of states

We have no quarrel with Khrennikov's opinion that the concept probability requires a more precise definition of the used probability distributions existence (non-negativity), and some additional consideration of the correspondence between this concept and the probability theory grounds. At the same time we believe that the requirement to never consider together the experimental statistics including different settings is too limiting. Such requirement may lead us to the statement that each single experiment is unique and inimitable (and what if a bus passes nearly during the experiment?), and that the statistics can't play a role in physics. We have to "do not throw the baby out with the bath water". In other words, we see an origin of such quantum inequalities in other (not so formal) reasons.

A number of authors came to idea that the crucial cause of Bell and Leggett inequalities violation is the state correlativity. Thus, author of [9] writes: "*<…> if the entanglement is present, then the non-locality of non-relativistic quantum mechanics at micro-level not only may, but has to be considered as basic (but not unique!) correlation source, and therefore as the source of a CHSH-inequalities violation <…> In order to talk surely on the possibility of the simultaneous a physical reality elements existence (or non-existence) we have to exclude all sources of potential correlations*". Particularly, the author of [9] argue that such existence (or non-existence) is unambiguously connected with existence (or non-existence) of the joint non-negative probability distributions. Indeed, a corresponding states cannot be "elementary" (in the sense of the Kolmogorov's definition) if and only if the existence area of these (generally non-elementary) are overlapped (or there is some "rupture" between them) that leads to appearance of a (positive or negative) correlation.

Let us add that in quantum mechanics the entangled systems may exist which are a superposition of several basic states. At the same time one can talk about a superposition (correlativity) too, for example, superposition of alternative classical trajectories, particularly while electrons or photons pass through a number of slits or during atom quantum walks. Such alternative superposition presents the base of the Feynmann path integrals formalism.

All above is surely confirmed (as we believe) by the given graphical illustrations of the unambiguous connection between correlativity degree and validity area of the inequalities (figures 1 – 4). From mathematician viewpoint it is not so important whether this correlativity between system parts is due the quantum world laws or, for example, to some special engineering communication algorithm between these parts.

However, if ones talks just about the quantum world laws, one should not disclaim that, e.g., in the EPR experiment an entangled photon pair *before* the measurement is in the superposition state and hasn't some unambiguously determined state, such the state appears only as result of the measurement on the although one of them. Furthermore, it is surely shown (see, e.g., [10]) that a measurement outcomes (at least, statistically) are conditioned by the analyzers settings which one could alter literally "at the last time moment" before the measurement. But the correlation value is just conditioned by this angle difference, and this means that measurement outcomes (final photon states) are determined only by the correlation set at the measuring time.

Unfortunately, the mathematical priority of the state correlativity does not explain what is a physical mechanism that creates such (particularly, non-local) the correlativity, and whether this mechanism requires a modern physical representation expansion. This problem is yet needed in its decision.

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